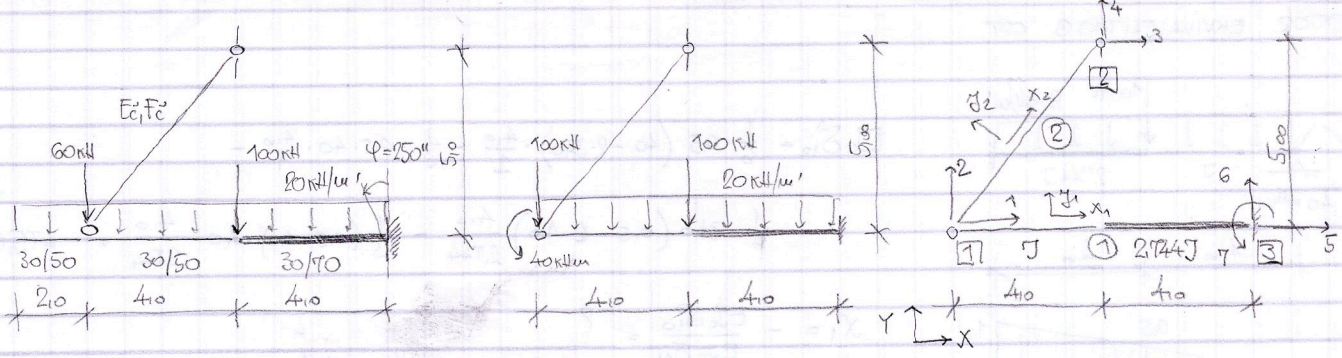


štap	i	k	λ	μ
1	1	3	1	0
2	1	2	0,6247	0,7808T

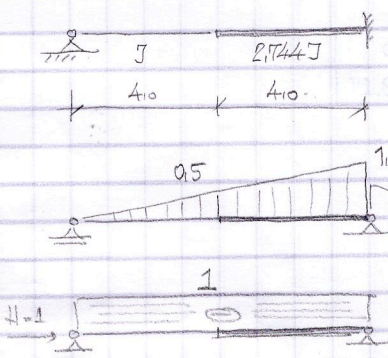
$E_b = 3 \cdot 10^7 kN/m^2$ $E_c = 21 \cdot 10^7 kN/m^2$; $F_c = 48 cm^2$

$J = \frac{30 \cdot 50^3}{12} = 312500 cm^4 = J_c$; $J = \frac{30 \cdot 70^3}{12} = 857500 cm^4 = 2,7442 J_c$

Matrice krutosti štapova

$F = 30 \cdot 50 = 1500 cm^2 = F_c$; $F = 30 \cdot 70 = 2100 cm^2 = 1,40 F_c$

- štap 1



$EJ_c \alpha_{11} = \frac{1}{3} \cdot 0,5^2 \cdot \frac{4,00}{1} + \frac{1}{6}\left[0,5 \cdot (2 \cdot 0,5 + 1,0) + 1,0 \cdot (0,5 + 2 \cdot 1,0)\right] \cdot \frac{4,00}{2,1744} = 1,1836T3469$

$EJ_c F_c \delta = 1^2 \cdot \frac{4,0}{1} + 1^2 \cdot \frac{4,0}{1,40} = 6,857142857 \; / \cdot \frac{J_c}{F_c}$

$EJ_c \delta = \frac{312500 \cdot 10^{-8}}{1500 \cdot 10^{-4}} = 0,1428571428$

$\frac{1}{\delta} = EJ_c \cdot T_{1,0}$

$d_{ig} = \frac{1}{\alpha_{11}} = EJ_c \cdot 0,84483$

$$K_1 = K_1^* = EJ_c \begin{bmatrix} 1 & 2 & 5 & 6 & 7 \\ T,100 & 0 & -T,10 & 0 & 0 \\ 0 & 0,0132 & 0 & -0,0132 & 0,1056 \\ -T,10 & 0 & T,10 & 0 & 0 \\ 0 & -0,0132 & 0 & 0,0132 & -0,1056 \\ 0 & 0,1056 & 0 & -0,1056 & 0,84483 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \\ 7 \end{matrix}$$

$EJ_c = 3 \cdot 10^7 \cdot 312500 \cdot 10^{-8} = 9375 kNm^2$

- štap 2

$$K_2 = \frac{E_c F_c}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{21 \cdot 10^7 \cdot 48 \cdot 10^{-4}}{6,403} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0,624T & 0,7808T & 0 & 0 \\ 0 & 0 & 0,624T & 0,7808T \end{bmatrix}$$

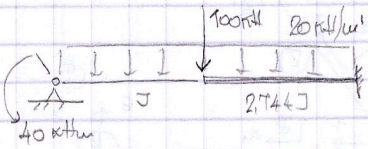
$$K_2^* = T_2^T K_2 T_2 = \begin{bmatrix} 0,624T & 0 \\ 0,7808T & 0 \\ 0 & 0,624T \\ 0 & 0,7808T \end{bmatrix} \frac{21 \cdot 48 \cdot 10^3}{6,403} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0,624T & 0,7808T & 0 & 0 \\ 0 & 0 & 0,624T & 0,7808T \end{bmatrix}$$

$$K_2^* = \frac{21 \cdot 48 \cdot 10^3}{6,403} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0,3903 & 0,487T8 & -0,3903 & -0,487T8 \\ & 0,6098 & -0,487T8 & -0,6098 \\ & & 0,3903 & 0,487T8 \\ & & & 0,6098 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

MATRICA KRUTOSTI SISTEMA

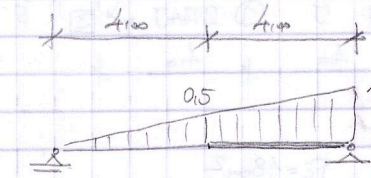
$$K^* = \begin{bmatrix} 1 & 2 \\ T1T693,4484 & T6T92,503517 \\ & 9T236,0 \end{bmatrix}$$

VEKTOR EKVIVALENTNOG OPT.

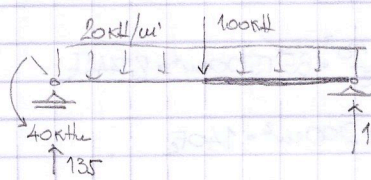


$$EJ_c\delta_{10} = \frac{1}{6}\cdot 0{,}5\cdot\left(40-2\cdot 340\right)\cdot\frac{4{,}10}{1} - \frac{1}{3}\cdot 0{,}5\cdot 40\cdot\frac{4{,}10}{1} -$$

$$-\frac{1}{6}\cdot 340\left(0{,}5+2\cdot 1{,}10\right)\frac{4{,}10}{2{,}744} - \frac{1}{3}\left(0{,}5+1{,}10\right)\cdot 40\cdot\frac{4{,}10}{2{,}744} = -475{,}66569\,48$$



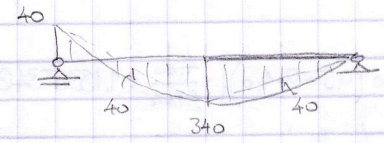
$$X_1 = -\frac{EJ_c\delta_{10}}{EJ_c\delta_{11}} = ?$$



$$Q_1 = Q_1^* = \begin{bmatrix} 1 & 2 & 5 & 6 & 7 \\ 0 & -89{,}27 & 0 & -170{,}73 & 365{,}84 \end{bmatrix}^T$$

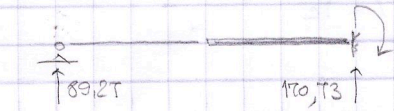
$$P_n^* = \begin{bmatrix} 1 & 2 \\ 0 & -100 \end{bmatrix}^T$$

$$S_n^* = \begin{bmatrix} 1 & 2 \\ 0 & -189{,}27 \end{bmatrix}^T$$



$$q_n^* = K_{nn}^{*-1} S_n^*$$

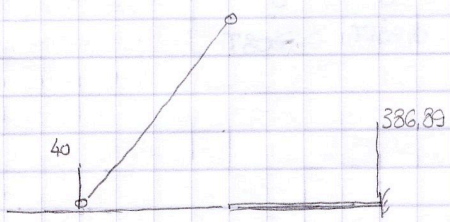
$$q_n^* = \begin{bmatrix} 111\,693{,}4484 & 76792{,}50351 \\ & 97236 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -189{,}27 \end{bmatrix}$$

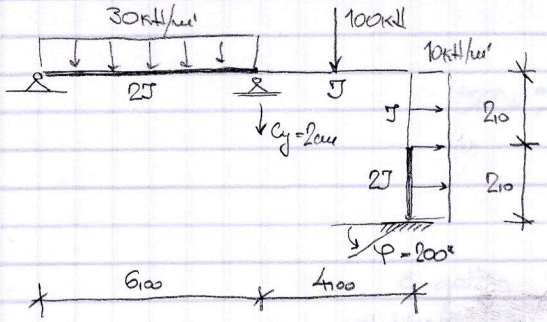


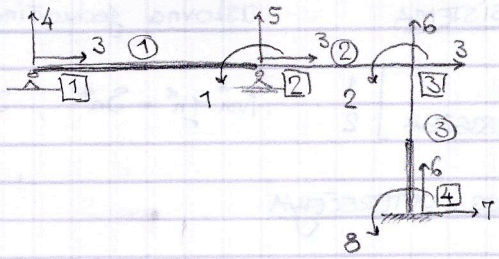
$$q_n^* = \begin{bmatrix} 0{,}00022\,7498 & -0{,}00212\,6168\,7 \end{bmatrix}^T$$

SILE NA KRAJEVIMA ŠTAPOVA $R_i = K_i T_i q_i^* - T_i Q_i^*$

$$R_M = 93750\begin{bmatrix} 1 & 2 & 5 & 6 & 7 \\ 7{,}10 & 0 & -7{,}10 & 0 & 0 \\ 0 & 0{,}0132 & 0 & -0{,}0132 & 0{,}1056 \\ -7{,}10 & 0 & 7{,}10 & 0 & 0 \\ 0 & -0{,}0132 & 0 & 0{,}0132 & -0{,}1056 \\ 0 & 0{,}1056 & 0 & -0{,}1056 & 0{,}84483 \end{bmatrix}\begin{bmatrix} 0{,}00022\,7498 \\ -0{,}00212\,6168\,7 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -89{,}27 \\ 0 \\ -170{,}73 \\ 365{,}84 \end{bmatrix} = \begin{bmatrix} 149{,}30 \\ 86{,}64 \\ -149{,}30 \\ 173{,}36 \\ -386{,}89 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$







b=0,30m ; h=0,60m; E=31500000 kN/m^2

	i	k	l	λ	μ	J
1	1	2	6,00	1	0	0,0108
2	2	3	4,00	1	0	0,0054
3	3	4	4,00	0	-1	

MATRICE KRUTOSTI ŠTAPOVA

- Štap 1 (tipa „g")

$$K_1 = K_1^* = \begin{bmatrix} \frac{3EJ}{l^3} & -\frac{3EJ}{l^3} & \frac{3EJ}{l^2} \\ -\frac{3EJ}{l^3} & \frac{3EJ}{l^3} & -\frac{3EJ}{l^2} \\ \frac{3EJ}{l^2} & -\frac{3EJ}{l^2} & \frac{3EJ}{l} \end{bmatrix} = \begin{matrix} 4 & 5 & 1 \\ \end{matrix}$$

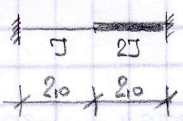
$$= \begin{bmatrix} 4T25 & -4T25 & 28350 \\ -4T25 & 4T25 & -28350 \\ 28350 & -28350 & 1T0100 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \end{matrix}$$

- Štap 2 (tipa „k")

$$K_2 = K_2^* = \begin{bmatrix} \frac{12EJ}{l^3} & \frac{6EJ}{l^2} & -\frac{12EJ}{l^3} & \frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & \frac{4EJ}{l} & -\frac{6EJ}{l^2} & \frac{2EJ}{l} \\ -\frac{12EJ}{l^3} & -\frac{6EJ}{l^2} & \frac{12EJ}{l^3} & -\frac{6EJ}{l^2} \\ -\frac{6EJ}{l^2} & \frac{2EJ}{l} & -\frac{6EJ}{l^2} & \frac{4EJ}{l} \end{bmatrix} = \begin{matrix} 5 & 1 & 6 & 2 \\ \end{matrix}$$

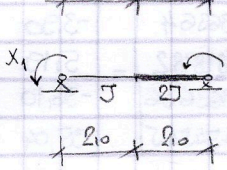
$$= \begin{bmatrix} 31893,75 & 63787,5 & -31893,75 & 63787,5 \\ 63787,5 & 1T0100 & -63787,5 & 85050 \\ -31893,75 & -63787,5 & 31893,75 & -63787,5 \\ -63787,5 & 85050 & -63787,5 & 1T0100 \end{bmatrix} \begin{matrix} 5 \\ 1 \\ 6 \\ 2 \end{matrix}$$

- Štap 3



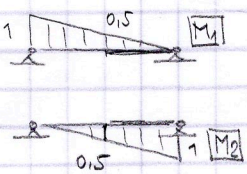
$J_c = J$

$$EJ_c\delta_{11} = \int_S M_1^2 \frac{J_c}{J} ds = \frac{1}{3}0,5^2 \cdot \frac{2,00}{2} + \frac{1}{6}\cdot\left[1,0\cdot(2\cdot 1,0+0,5)+0,5\cdot(1,0+2\cdot 0,5)\right]\cdot 2,00 = 1,25$$



$$EJ_c\delta_{12} = \int_S M_1 M_2 \frac{J_c}{J} ds = -\frac{1}{6}0,5\cdot(2\cdot 0,5+1,0)\cdot\frac{2,00}{2} - \frac{1}{6}0,5\cdot(2\cdot 0,5+1,0)\cdot 2,00 = -0,50$$

$$EJ_c\delta_{22} = \int_S M_2^2 \frac{J_c}{J} ds = \frac{1}{3}0,5^2\cdot 2,00 + \frac{1}{6}\cdot\left[0,5\cdot(2\cdot 0,5+1,0)+1,0(0,5+2\cdot 1,0)\right]\cdot\frac{2,00}{2} = 0,75$$



matrica fleksibilnosti

$$f = \frac{1}{EJ_c}\begin{bmatrix} 1,25 & -0,5 \\ -0,5 & 0,75 \end{bmatrix}$$

Lokalna matrica krutosti

$$K_0 = f^{-1} = EJ_c \begin{bmatrix} 1,090909091 & 0,727272727 \\ 0,727272727 & 1,818181818 \end{bmatrix}$$

matrica ravnoteže

$$C = \begin{bmatrix} 1/l & 1 & -1/l & 0 \\ 1/l & 0 & -1/l & 1 \end{bmatrix} = \begin{bmatrix} 0,25 & 1 & -0,25 & 0 \\ 0,25 & 0 & -0,25 & 1 \end{bmatrix}$$

$$K_3 = C^T K_0 C = K_3^* = \begin{matrix} 3 & 2 & 7 & 8 \\ \end{matrix}$$

$$= \begin{bmatrix} 46390,90908 & 77318,18181 & -46390,90908 & 108245,454526 \\ 77318,18181 & 185563,636364 & -77318,18181 & 123700,090909 \\ -46390,90908 & -77318,18181 & 46390,90908 & -108245,454526 \\ 108245,454526 & 123700,090909 & -108245,454526 & 309272,727272418 \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 7 \\ 8 \end{matrix}$$

MATRICA KRUTOSTI SISTEMA

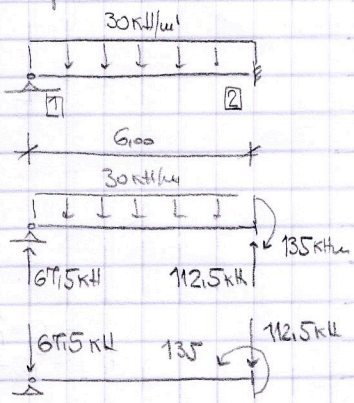
$$K_{nn}^{*} = \begin{bmatrix} 340200 & 85050 \\ 85050 & 355663{,}636364 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Uslovna jednačina

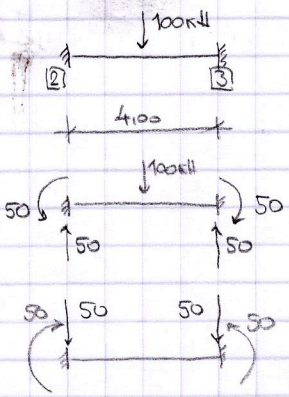
$$K_{nn}^{*} q_n^{*} = S_n^{*}, \quad S_n^{*} = Q_n^{*} + P_n^{*}$$

VEKTOR EKVIVALENTOG OPTEREĆENJA

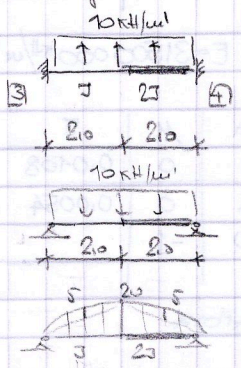
- štap 1



- štap 2



- štap 3



$$EJ_c\delta_{10} = \int_S M_1 M_0 \frac{J_c}{J} ds =$$

$$= +\frac{1}{3}(1{,}0+0{,}5)\cdot 5\cdot 2{,}0 +$$

$$+\frac{1}{6}(1{,}0+2\cdot 0{,}5)\cdot 20\cdot 2{,}0 +$$

$$+\frac{1}{3}\,0{,}5\cdot 20\cdot \frac{2{,}0}{2} +$$

$$+\frac{1}{3}\cdot 0{,}5\cdot 5\cdot \frac{2{,}00}{2} = +22{,}5$$

$$EJ_c\delta_{20} = -\frac{1}{3}\cdot 0{,}5\cdot 20\cdot 2{,}00 -$$

$$+\frac{1}{3}\cdot 0{,}5\cdot 5\cdot 2{,}00 - \frac{1}{6}\,20\left(2\cdot 0{,}5+1{,}0\right)\cdot\frac{2{,}0}{2} - \frac{1}{3}\cdot(0{,}5+1{,}0)\cdot 5{,}0\cdot\frac{2{,}0}{2} =$$

$$= -17{,}50$$

$$\left.\begin{aligned} 1{,}25X_1 - 0{,}5X_2 + 22{,}5 = 0 \\ -0{,}5X_1 + 0{,}75X_2 - 17{,}50 = 0 \end{aligned}\right\} \quad \begin{aligned} X_1 &= -24{,}1818182 \\ X_2 &= 15{,}454545 \end{aligned}$$

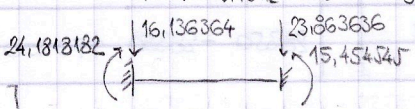
$$Q_1 = Q_1^{*} = \begin{bmatrix} -67{,}50 & -112{,}5 & 135 \end{bmatrix}^T \qquad \begin{matrix} 4 & 5 & 1 \end{matrix}$$

$$Q_2 = Q_2^{*} = \begin{bmatrix} -50 & -50 & -50 & 50 \end{bmatrix}^T \qquad \begin{matrix} 5 & 1 & 6 & 2 \end{matrix}$$

$$Q_n^{*} = \begin{bmatrix} 85 & 74{,}1818182 \end{bmatrix}^T \qquad P_n^{*} = 0$$

$$q_n^{*} = \begin{bmatrix} 0{,}000003126348663 & -0{,}000000747605151 \\ -0{,}000000747605151 & 0{,}000002990042046 \end{bmatrix} \begin{bmatrix} 85 \\ 74{,}1818182 \end{bmatrix}$$

$$q_n^{*} = \begin{bmatrix} 0{,}000210280929 6 & 0{,}000158288392 1 \end{bmatrix}^T \qquad \begin{matrix} 1 & 2 \end{matrix}$$



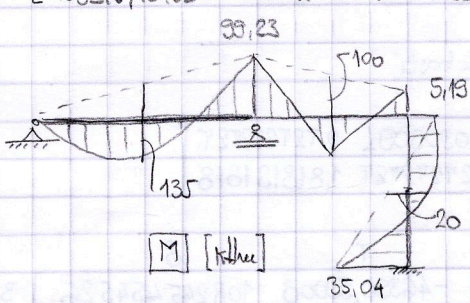
$$Q_3 = Q_3^{*} = \begin{bmatrix} 16{,}136364 & 24{,}1818182 & 23{,}863636 & -15{,}454545 \end{bmatrix}^T \qquad \begin{matrix} 3 & 2 & 7 & 8 \end{matrix}$$

SILE NA KRAJEVIMA STAPA $R_i = K_i q_i - Q_i$

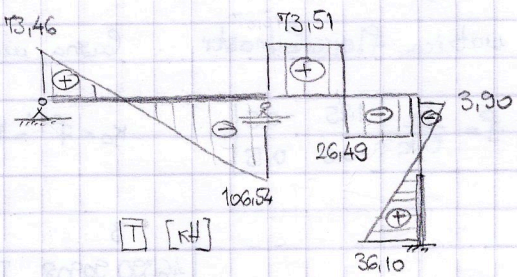
$$R_1 = \begin{bmatrix} 4725 & -4725 & 28350 \\ -4725 & 4725 & -28350 \\ 28350 & -28350 & 170100 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0{,}000210280929 6 \end{bmatrix} - \begin{bmatrix} -67{,}50 \\ -112{,}5 \\ 135 \end{bmatrix} = \begin{bmatrix} 73{,}46 \\ 106{,}54 \\ -99{,}23 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \end{matrix}$$

$$R_2 = \begin{bmatrix} 31893{,}75 & 63787{,}5 & -31893{,}75 & 63787{,}5 \\ 63787{,}5 & 170100 & -63787{,}5 & 85050 \\ -31893{,}75 & -63787{,}5 & 31893{,}75 & -63787{,}5 \\ 63787{,}5 & 85050 & -63787{,}5 & 170100 \end{bmatrix} \begin{matrix} 5 \\ 1 \\ 6 \\ 2 \end{matrix} \begin{bmatrix} 0 \\ 0{,}000210280929 6 \\ 0 \\ 0{,}000158288392 1 \end{bmatrix} - \begin{bmatrix} -50 \\ -50 \\ -50 \\ 50 \end{bmatrix} = \begin{bmatrix} 73{,}51 \\ 99{,}23 \\ 26{,}49 \\ -5{,}19 \end{bmatrix} \begin{matrix} 5 \\ 1 \\ 6 \\ 2 \end{matrix}$$

$$R_3 = \begin{bmatrix} 46390{,}90908 & 77318{,}18181 & -46390{,}90908 & 108245{,}454526 \\ 77318{,}18181 & 185563{,}636364 & -77318{,}18181 & 123709{,}090909 \\ -46390{,}90908 & -77318{,}18181 & 46390{,}90908 & -108245{,}454526 \\ 108245{,}454526 & 123709{,}090909 & -108245{,}454526 & 309272{,}727273 \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 7 \\ 8 \end{matrix} \begin{bmatrix} 0 \\ 0{,}000158288392 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 16{,}136364 \\ 24{,}1818182 \\ 23{,}863636 \\ -15{,}454545 \end{bmatrix} = \begin{bmatrix} -3{,}90 \\ 5{,}19 \\ -36{,}10 \\ 35{,}04 \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 7 \\ 8 \end{matrix}$$



M [kNm]

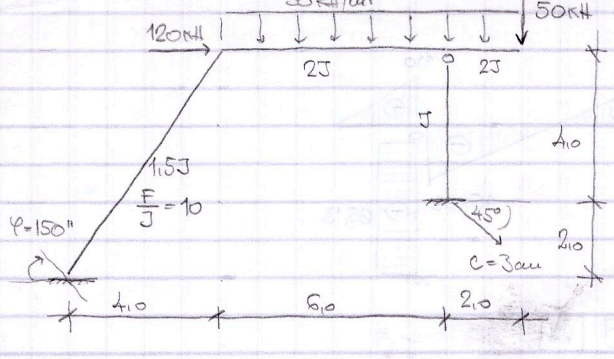


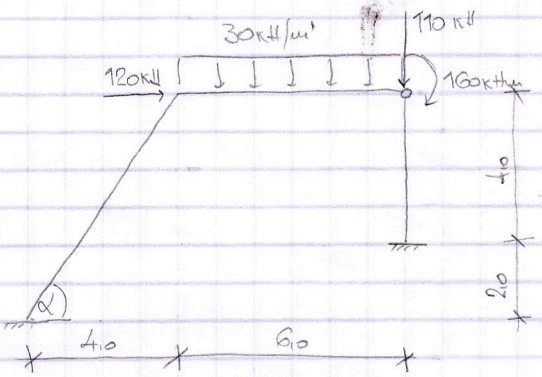
T [kN]

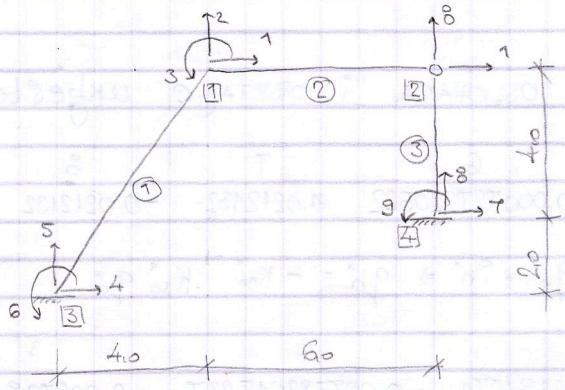
BLANKET 21.08.2015.

Približna metoda deformacije

$EJ = 100\,000\ kNm^2$







Matrice krutosti štapova

- Štap 1 $\ell_1 = 7{,}211\,m$

$$K_1 = \begin{bmatrix} \frac{EF}{\ell} & 0 & 0 & -\frac{EF}{\ell} & 0 & 0 \\ 0 & \frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} & 0 & -\frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} \\ 0 & \frac{6EJ}{\ell^2} & \frac{4EJ}{\ell} & 0 & -\frac{6EJ}{\ell^2} & \frac{2EJ}{\ell} \\ -\frac{EF}{\ell} & 0 & 0 & \frac{EF}{\ell} & 0 & 0 \\ 0 & -\frac{12EJ}{\ell^3} & -\frac{6EJ}{\ell^2} & 0 & \frac{12EJ}{\ell^3} & -\frac{6EJ}{\ell^2} \\ 0 & \frac{6EJ}{\ell^2} & \frac{2EJ}{\ell} & 0 & -\frac{6EJ}{\ell^2} & \frac{4EJ}{\ell} \end{bmatrix} = EJ \begin{bmatrix} 1{,}3867 & 0 & 0 & -1{,}3867 & 0 & 0 \\ 0 & 0{,}048003 & 0{,}17307 & 0 & -0{,}048003 & 0{,}17307 \\ 0 & 0{,}17307 & 0{,}83205 & 0 & -0{,}17307 & 0{,}416025 \\ -1{,}3867 & 0 & 0 & 1{,}3867 & 0 & 0 \\ 0 & -0{,}048003 & -0{,}17307 & 0 & 0{,}048003 & -0{,}17307 \\ 0 & 0{,}17307 & 0{,}416025 & 0 & -0{,}17307 & 0{,}83205 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} \cos\alpha & \sin\alpha & & & & \\ -\sin\alpha & \cos\alpha & & & & \\ & & 1 & & & \\ & & & \cos\alpha & \sin\alpha & \\ & & & -\sin\alpha & \cos\alpha & \\ & & & & & 1 \end{bmatrix} = \begin{bmatrix} 0{,}554 & 0{,}8320503 & & & & \\ -0{,}8320503 & 0{,}554 & & & & \\ & & 1 & & & \\ & & & 0{,}554 & 0{,}8320503 & \\ & & & -0{,}8320503 & 0{,}554 & \\ & & & & & 1 \end{bmatrix}$$

$$K_1^* = T_1^T K_1 T_1 = EJ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0{,}459925 & 0{,}617883 & -0{,}1440088 & -0{,}459925 & -0{,}617883 & -0{,}1440088 \\ 0{,}617883 & 0{,}974828 & 0{,}0960058 & -0{,}617883 & -0{,}974828 & 0{,}0960058 \\ -0{,}1440088 & 0{,}0960058 & 0{,}83205 & 0{,}1440088 & -0{,}0960058 & 0{,}416025 \\ -0{,}459925 & -0{,}617883 & 0{,}1440088 & 0{,}459925 & 0{,}617883 & 0{,}1440088 \\ -0{,}617883 & -0{,}974828 & -0{,}0960058 & 0{,}617883 & 0{,}974828 & -0{,}0960058 \\ -0{,}1440088 & 0{,}0960058 & 0{,}416025 & 0{,}1440088 & -0{,}0960058 & 0{,}83205 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{array}$$

- Štap 2 $\ell_2 = 6{,}100\,m$

$$K_2 = \frac{EJ}{\ell^3} \begin{bmatrix} 3 & 3\ell & -3 \\ 3\ell & 3\ell^2 & -3\ell \\ -3 & -3\ell & 3 \end{bmatrix} = EJ \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 8 \end{array} \\ \begin{bmatrix} 0{,}027778 & 0{,}1666 & -0{,}027778 \\ 0{,}1666 & 1{,}0 & -0{,}1666 \\ -0{,}027778 & -0{,}1666 & 0{,}027778 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \\ 8 \end{array} = K_2^*$$

- Štap 3 $\ell_3 = 4{,}100\,m$

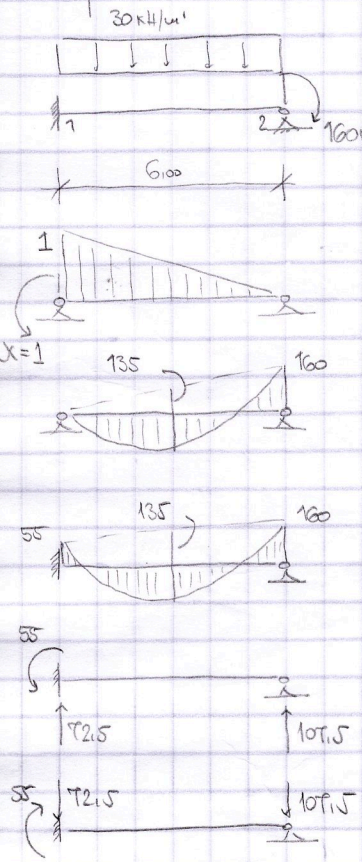
$$K_3 = \frac{EJ}{\ell^3} \begin{bmatrix} 3 & -3 & 3\ell \\ -3 & 3 & -3\ell \\ 3\ell & -3\ell & 3\ell^2 \end{bmatrix} = EJ \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 7 & 9 \end{array} \\ \begin{bmatrix} 0{,}04685 & -0{,}04685 & 0{,}185 \\ -0{,}04685 & 0{,}04685 & -0{,}185 \\ 0{,}185 & -0{,}185 & 0{,}75 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 7 \\ 9 \end{array} = K_3^*$$

$K^* = EJ$

1	2	3						
0,5068	0,617883	0,1440088	-0,459925	-0,617883	0,1440088	-0,046875	0	0,1875
	1,002606	0,0706642	-0,617883	-0,974828	-0,0960058	0	-0,02777	0
		1,83205	-0,1440088	0,0960058	0,416025	0	-0,16667	0

Vektor ekvivalentnog opterećenja

- Štap 2



$EJ\delta_{11}^0 = \frac{1}{3} \cdot 1^2 \cdot 6,0 = 2,0$

$EJ\delta_{10}^0 = \frac{1}{6} \cdot 1 \cdot 160 \cdot 6,0 - \frac{1}{3} \cdot 1 \cdot 135 \cdot 6,0 = -110$

$X = \frac{-EJ\delta_{10}^0}{EJ\delta_{11}} = \frac{110}{2} = 55\,kNm$

$T_1 = \frac{30 \cdot 6,0}{2} - \frac{160 - 55}{6,0} = T2,5\,kN$

$T_2 = -\frac{30 \cdot 6,0}{2} - \frac{160 - 55}{6,0} = -10T,5\,kN$

$Q_2 = Q_2^* = [\underset{2}{-T2,5} \quad \underset{3}{-55} \quad \underset{8}{-10T,50}]^T$

$Q^* = [\underset{1}{0} \quad \underset{2}{-T2,50} \quad \underset{3}{-55} \quad \underset{4}{0} \quad \underset{5}{0} \quad \underset{6}{0} \quad \underset{T}{0} \quad \underset{8}{-10T,50} \quad \underset{9}{0}]$

$P^* = [\underset{1}{120} \quad \underset{2}{0} \quad \underset{3}{0} \quad \underset{4}{0} \quad \underset{5}{0} \quad \underset{6}{0} \quad \underset{T}{0} \quad \underset{8}{-110} \quad \underset{9}{0}]$

$S_n^* = Q_n^* + P_n^* = [\underset{1}{120} \quad \underset{2}{-T2,50} \quad \underset{3}{-55}]$

Ustovne jednačine - proračun nepoznatih generalisanih pomeranja

$K_{nn}^* q_n^* = S_n^* \rightarrow q_n^* = K_{nn}^{*-1} S_n^*$

$q_n^* = [\underset{1}{1389,52908} \quad \underset{2}{-921,336926} \quad \underset{3}{-103,T0834\,8}]^T \cdot \frac{1}{EJ}$

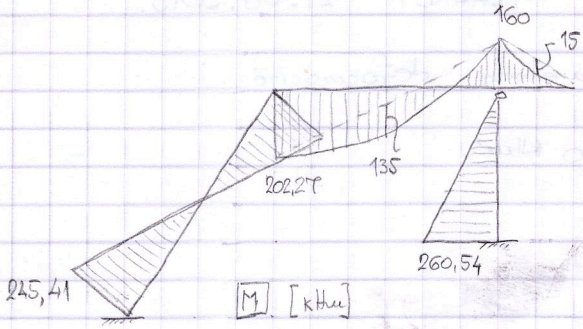
Sile na krajevima štapova $R_i = K_i T_i q_i^* - T_i Q_i^*$

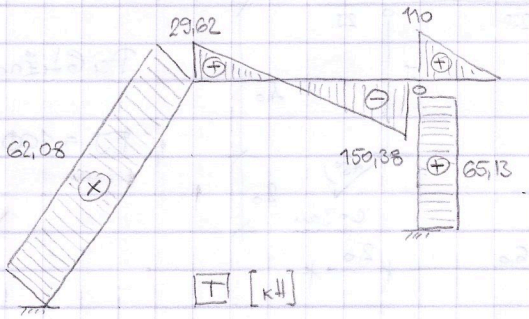
Štap 1

$q_1^* = [\underset{4}{0} \quad \underset{5}{0} \quad \underset{6}{0} \quad \underset{1}{1389,52908} \quad \underset{2}{-921,336926} \quad \underset{3}{-103,T0834\,8}]^T \frac{1}{EJ}$; $q_2^* = [\underset{2}{-921,336926} \quad \underset{3}{-103,T0834\,8} \quad \underset{8}{0}]^T \frac{1}{EJ}$

$R_1 = \begin{bmatrix} -5,T9 \\ 62,08 \\ 245,41 \\ 5,T9 \\ -62,08 \\ 202,27 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{matrix}$ $R_2 = \begin{bmatrix} 29,62 \\ -202,27 \\ 150,38 \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 8 \end{matrix}$ $R_3 = \begin{bmatrix} 65,13 \\ -65,13 \\ 260,54 \end{bmatrix} \begin{matrix} 1 \\ 7 \\ 9 \end{matrix}$

$q_3^* = [\underset{1}{1389,52908} \quad \underset{T}{0} \quad \underset{9}{0}]^T \frac{1}{EJ}$





Pomeranje OSLONACA i OBRTANJE uklještenja

$$q_0^* = \begin{bmatrix} 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0{,}000727220522 & 0{,}0212132 & -0{,}0212132 & 0 \end{bmatrix}^T$$

$$K_{nn}^* q_n^* + K_{s0}^* q_0^* = S_n^* \Rightarrow q_n^* = -K_{nn}^{*-1} K_{s0}^* q_0^*$$

$$q_n^* = \begin{bmatrix} 1 & 2 & 3 \\ 0{,}011775228457 & -0{,}007582615237 & -0{,}002728125893 \end{bmatrix}^T$$

Sile na krajevima štapova $R_i = K_i T_i q_i^*$

$$q_1^* = \begin{bmatrix} 4 & 5 & 6 & 1 & 2 & 3 \\ 0 & 0 & 0{,}000727220522 & 0{,}011775228457 & -0{,}007582615237 & -0{,}002728125893 \end{bmatrix}^T$$

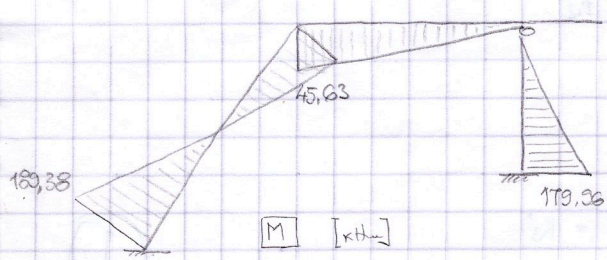
$$R_1 = \begin{bmatrix} -30{,}87 & 4 \\ 32{,}59 & 5 \\ 189{,}38 & 6 \\ 30{,}87 & 1 \\ -32{,}59 & 2 \\ 45{,}63 & 3 \end{bmatrix}$$

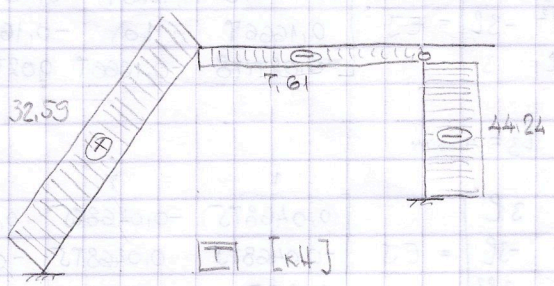
$$q_2^* = \begin{bmatrix} 2 & 3 & 8 \\ -0{,}007582615237 & -0{,}002728125893 & -0{,}0212132 \end{bmatrix}^T$$

$$R_2 = \begin{bmatrix} -7{,}61 & 2 \\ -45{,}63 & 3 \\ 7{,}61 & 8 \end{bmatrix}$$

$$q_3^* = \begin{bmatrix} 1 & 7 & 9 \\ 0{,}011775228457 & 0{,}0212132 & 0 \end{bmatrix}^T$$

$$R_3 = \begin{bmatrix} -44{,}24 & 1 \\ 44{,}24 & 7 \\ -176{,}96 & 9 \end{bmatrix}$$





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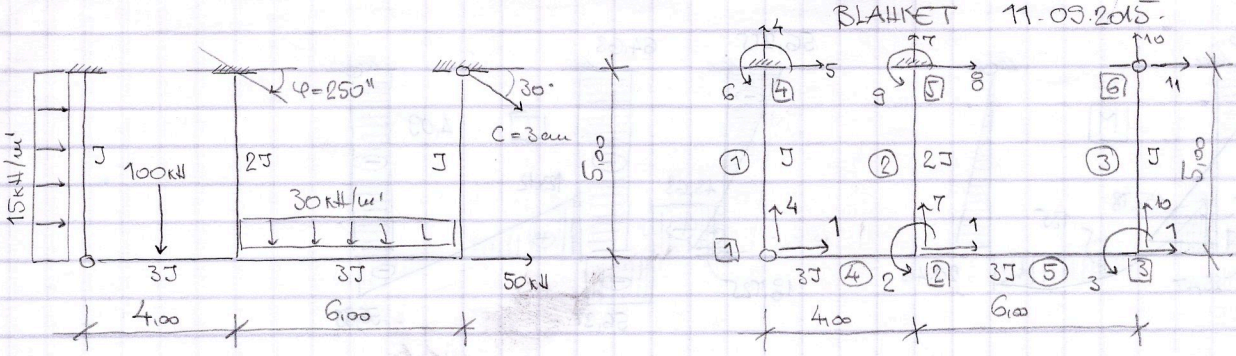


Matrice krutosti štapova:

- Štap 1 $l = 5{,}00$ m

$$K_1 = \begin{bmatrix} \frac{3EJ}{l^3} & \frac{3EJ}{l^2} & -\frac{3EJ}{l^3} \\ \frac{3EJ}{l^2} & \frac{3EJ}{l} & -\frac{3EJ}{l^2} \\ -\frac{3EJ}{l^3} & -\frac{3EJ}{l^2} & \frac{3EJ}{l^3} \end{bmatrix} = EJ \begin{bmatrix} 0{,}024 & 0{,}12 & -0{,}024 \\ 0{,}12 & 0{,}6 & -0{,}12 \\ -0{,}024 & -0{,}12 & 0{,}024 \end{bmatrix} = K_1^* \quad \begin{matrix} 5 \\ 6 \\ 1 \end{matrix}$$

(5 6 1)

- Štap 2 $l = 5{,}00$ m

$$K_2 = \begin{bmatrix} \frac{12EJ}{l^3} & \frac{6EJ}{l^2} & -\frac{12EJ}{l^3} & \frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & \frac{4EJ}{l} & -\frac{6EJ}{l^2} & \frac{2EJ}{l} \\ -\frac{12EJ}{l^3} & -\frac{6EJ}{l^2} & \frac{12EJ}{l^3} & -\frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & \frac{2EJ}{l} & -\frac{6EJ}{l^2} & \frac{4EJ}{l} \end{bmatrix} = EJ \begin{bmatrix} 0{,}192 & 0{,}48 & -0{,}192 & 0{,}48 \\ 0{,}48 & 1{,}60 & -0{,}48 & 0{,}80 \\ -0{,}192 & -0{,}48 & 0{,}192 & -0{,}48 \\ 0{,}48 & 0{,}80 & -0{,}48 & 1{,}60 \end{bmatrix} = K_2^* \quad \begin{matrix} 8 \\ 9 \\ 1 \\ 2 \end{matrix}$$

(8 9 1 2)

- Štap 3 $l = 5{,}10$ m

$$K_3 = \begin{bmatrix} \frac{3EJ}{l^3} & -\frac{3EJ}{l^3} & \frac{3EJ}{l^2} \\ -\frac{3EJ}{l^3} & \frac{3EJ}{l^3} & -\frac{3EJ}{l^2} \\ \frac{3EJ}{l^2} & -\frac{3EJ}{l^2} & \frac{3EJ}{l} \end{bmatrix} = EJ \begin{bmatrix} 0{,}024 & -0{,}024 & 0{,}12 \\ -0{,}024 & 0{,}024 & -0{,}12 \\ 0{,}12 & -0{,}12 & 0{,}60 \end{bmatrix} = K_3^* \quad \begin{matrix} 11 \\ 1 \\ 3 \end{matrix}$$

(11 1 3)

- Štap 4 $l = 4{,}00$ m

$$K_4 = \begin{bmatrix} \frac{3EJ}{l^3} & -\frac{3EJ}{l^3} & \frac{3EJ}{l^2} \\ -\frac{3EJ}{l^3} & \frac{3EJ}{l^3} & -\frac{3EJ}{l^2} \\ \frac{3EJ}{l^2} & -\frac{3EJ}{l^2} & \frac{3EJ}{l} \end{bmatrix} = EJ \begin{bmatrix} 0{,}140625 & -0{,}140625 & 0{,}5625 \\ -0{,}140625 & 0{,}140625 & -0{,}5625 \\ 0{,}5625 & -0{,}5625 & 2{,}25 \end{bmatrix} = K_4^* \quad \begin{matrix} 4 \\ 7 \\ 2 \end{matrix}$$

(4 7 2)

- Štap 5 $l = 6{,}00$ m

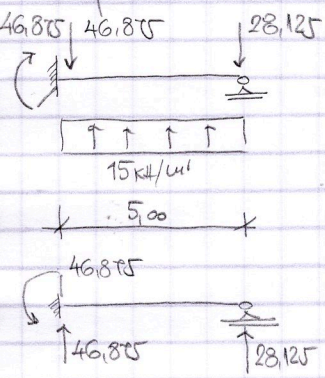
$$K_5 = \begin{bmatrix} \frac{12EJ}{l^3} & \frac{6EJ}{l^2} & -\frac{12EJ}{l^3} & \frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & \frac{4EJ}{l} & -\frac{6EJ}{l^2} & \frac{2EJ}{l} \\ -\frac{12EJ}{l^3} & -\frac{6EJ}{l^2} & \frac{12EJ}{l^3} & -\frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & \frac{2EJ}{l} & -\frac{6EJ}{l^2} & \frac{4EJ}{l} \end{bmatrix} = EJ \begin{bmatrix} 0{,}166667 & 0{,}50 & -0{,}166667 & 0{,}50 \\ 0{,}50 & 2{,}10 & -0{,}50 & 1{,}10 \\ -0{,}166667 & -0{,}50 & 0{,}166667 & -0{,}50 \\ 0{,}50 & 1{,}10 & -0{,}50 & 2{,}10 \end{bmatrix} = K_4^* \quad \begin{matrix} 7 \\ 2 \\ 10 \\ 3 \end{matrix}$$

(7 2 10 3)

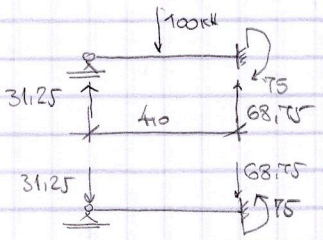
$$K^* = EJ \begin{bmatrix} 0{,}24 & -0{,}48 & -9{,}12 & 0 & -0{,}024 & -0{,}12 & 0 & -0{,}192 & -0{,}48 & 0 & -0{,}024 \\ -0{,}48 & 5{,}85 & 1{,}0 & 0{,}5625 & 0 & 0 & -0{,}0625 & 0{,}48 & 0{,}80 & -0{,}50 & 0 \\ -0{,}12 & 1{,}0 & 2{,}60 & 0 & 0 & 0 & 0{,}50 & 0 & 0 & -0{,}50 & 0{,}12 \end{bmatrix}$$

VEKTOR EKVIVALENTNOG OPTEREĆENJA

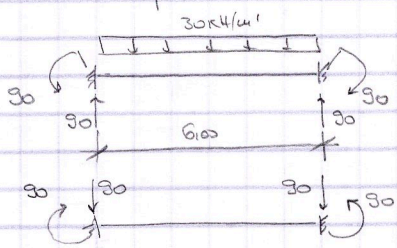
- štap 1



- štap 4



- štap 5



$$Q_4 = Q_4^* = \begin{bmatrix} 4 & 7 & 2 \\ -31{,}25 & -68{,}75 & 75 \end{bmatrix}^T \qquad Q_5^* = Q_5 = \begin{bmatrix} 7 & 2 & 10 & 3 \\ -90 & -90 & -90 & 90 \end{bmatrix}^T$$

$$Q_1^* = Q_1 = \begin{bmatrix} 5 & 6 & 1 \\ 46{,}875 & 46{,}875 & 28{,}125 \end{bmatrix}^T$$

$$Q^* = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 28{,}125 & -15 & 90 & -31{,}25 & 46{,}875 & 46{,}875 & -168{,}75 & 0 & 0 & -90 & 0 \end{bmatrix}^T$$

$$P^* = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$S_n^* = Q_n^* + P_n^* = \begin{bmatrix} 28{,}125 & -15 & 90 \end{bmatrix}^T + \begin{bmatrix} 50 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 78{,}125 & -15 & 90 \end{bmatrix}^T$$

Uslovne jednačine – proračun nepoznatih parametara pomeranja

$$K_{nn}^* q_n^* = S_n^* \Rightarrow q_n^* = K_{nn}^{*-1} S_n^*$$

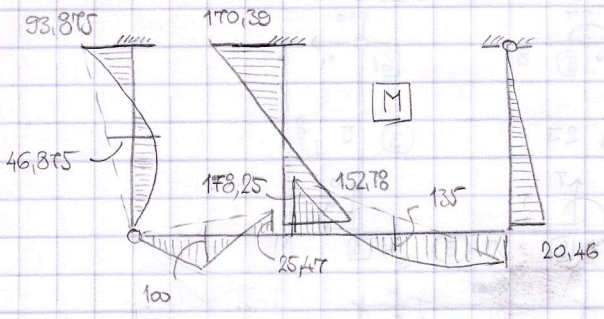
$$q_n^* = \frac{1}{EJ} \begin{bmatrix} 0{,}24 & -0{,}48 & -0{,}12 \\ -0{,}48 & 5{,}85 & 1{,}0 \\ -0{,}12 & 1{,}0 & 2{,}60 \end{bmatrix}^{-1} \begin{bmatrix} 78{,}125 \\ -15 \\ 90 \end{bmatrix} = \frac{1}{EJ} \begin{bmatrix} 391{,}6576 \\ 22{,}0119 \\ 44{,}2258 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

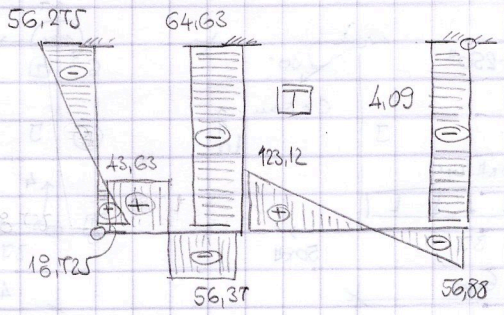
SILE NA KRAJEVIMA ŠTAPOVA $R_i = K_i T_i q_i^* - T_i Q_i^*$

$$q_1^* = \begin{bmatrix} 0 \\ 0 \\ 391{,}6576 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 1 \end{matrix} \quad \frac{1}{EJ} \quad R_1 = \begin{bmatrix} -56{,}275 \\ -93{,}875 \\ -18{,}725 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 1 \end{matrix} \qquad q_2^* = \begin{bmatrix} 0 \\ 0 \\ 391{,}6576 \\ 22{,}0119 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 1 \\ 2 \end{matrix} \quad \frac{1}{EJ} \quad R_2 = \begin{bmatrix} -64{,}63 \\ -170{,}39 \\ 64{,}63 \\ -152{,}76 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 1 \\ 2 \end{matrix}$$

$$q_3^* = \begin{bmatrix} 0 \\ 391{,}6576 \\ 44{,}2258 \end{bmatrix} \begin{matrix} 11 \\ 1 \\ 3 \end{matrix} \quad \frac{1}{EJ} \quad R_3 = \begin{bmatrix} -4{,}09 \\ 4{,}09 \\ -20{,}46 \end{bmatrix} \begin{matrix} 11 \\ 1 \\ 3 \end{matrix} \qquad q_4^* = \begin{bmatrix} 0 \\ 0 \\ 22{,}0119 \end{bmatrix} \begin{matrix} 4 \\ 7 \\ 2 \end{matrix} \quad \frac{1}{EJ} \quad R_4 = \begin{bmatrix} 43{,}63 \\ 56{,}37 \\ -25{,}47 \end{bmatrix} \begin{matrix} 4 \\ 7 \\ 2 \end{matrix}$$

$$q_5^* = \begin{bmatrix} 0 \\ 22{,}0119 \\ 0 \\ 44{,}2258 \end{bmatrix} \begin{matrix} 7 \\ 2 \\ 10 \\ 3 \end{matrix} \quad \frac{1}{EJ} \quad R_5 = \begin{bmatrix} 123{,}12 \\ 176{,}25 \\ 56{,}88 \\ 20{,}46 \end{bmatrix} \begin{matrix} 7 \\ 2 \\ 10 \\ 3 \end{matrix}$$





Pomeranje oslonaca i obtežanje uklještenja

$$q_0^* = [0\ 0\ 0\ 0\ 0\ -0{,}001212034203\ -0{,}015\ 0{,}025980762]^T$$
(4 5 6 7 8 9 10 11)

$$K_{nn}^* q_n^* + K_{no}^* q_0^* = S_n^* \Rightarrow q_n^* = -K_{nn}^{*-1} K_{no}^* q_0^*$$
(1 2 3)

$$q_n^* = [-0{,}003212147701256\ -0{,}000702644618211\ -0{,}003961773251766 9]^T$$

Sile na krajevima štapova $R_i = K_i T_i q_i^*$ Neka je $J = \frac{1}{1000}\,m^4$ $EJ = 30000\ kNm^2$

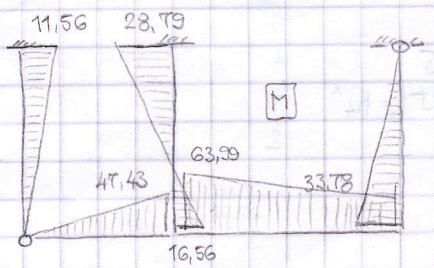
$$q_1^* = \begin{bmatrix} 0 \\ 0 \\ -0{,}00321215 \end{bmatrix};\quad R_1 = EJ \begin{bmatrix} 0{,}000077009154 5 \\ 0{,}00038545 77 \\ -0{,}000077009154 5 \end{bmatrix} = \begin{bmatrix} 2{,}31 \\ 11{,}56 \\ -2{,}31 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 1 \end{matrix}$$

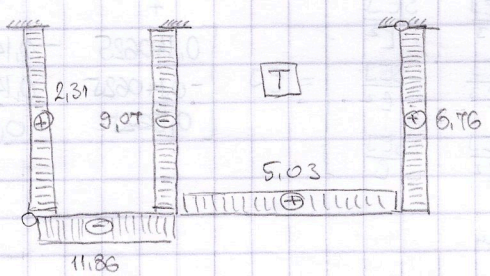
$$q_2^* = \begin{bmatrix} 0 \\ -0{,}001212034 \\ -0{,}00321215 \\ -0{,}000702645 \end{bmatrix};\quad R_2 = EJ \begin{bmatrix} -0{,}000302313 12 \\ -0{,}000959538 4 \\ 0{,}000302313 12 \\ -0{,}000552027 2 \end{bmatrix} = \begin{bmatrix} -9{,}07 \\ -28{,}79 \\ 9{,}07 \\ -16{,}56 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 1 \\ 2 \end{matrix}$$

$$q_3^* = \begin{bmatrix} 0{,}025980762 \\ -0{,}00321215 \\ -0{,}00396177 \end{bmatrix};\quad R_3 = EJ \begin{bmatrix} 0{,}000225221931 \\ -0{,}000225221931 \\ 0{,}001126109654 \end{bmatrix} = \begin{bmatrix} 6{,}76 \\ -6{,}76 \\ 33{,}78 \end{bmatrix} \begin{matrix} 11 \\ 1 \\ 3 \end{matrix}$$

$$q_4^* = \begin{bmatrix} 0 \\ 0 \\ -0{,}000702644 \end{bmatrix};\quad R_4 = EJ \begin{bmatrix} -0{,}000395237 6 \\ 0{,}000395237 6 \\ -0{,}001580950 4 \end{bmatrix} = \begin{bmatrix} -11{,}86 \\ 11{,}86 \\ -47{,}43 \end{bmatrix} \begin{matrix} 4 \\ 7 \\ 2 \end{matrix}$$

$$q_5^* = \begin{bmatrix} 0 \\ -0{,}000702645 \\ -0{,}015 \\ -0{,}003961773 \end{bmatrix};\quad R_5 = EJ \begin{bmatrix} 0{,}000167816432 \\ 0{,}002132977824 \\ -0{,}000167816432 \\ -0{,}001126109653 \end{bmatrix} = \begin{bmatrix} 5{,}03 \\ 63{,}99 \\ -5{,}03 \\ -33{,}78 \end{bmatrix} \begin{matrix} 7 \\ 2 \\ 10 \\ 3 \end{matrix}$$





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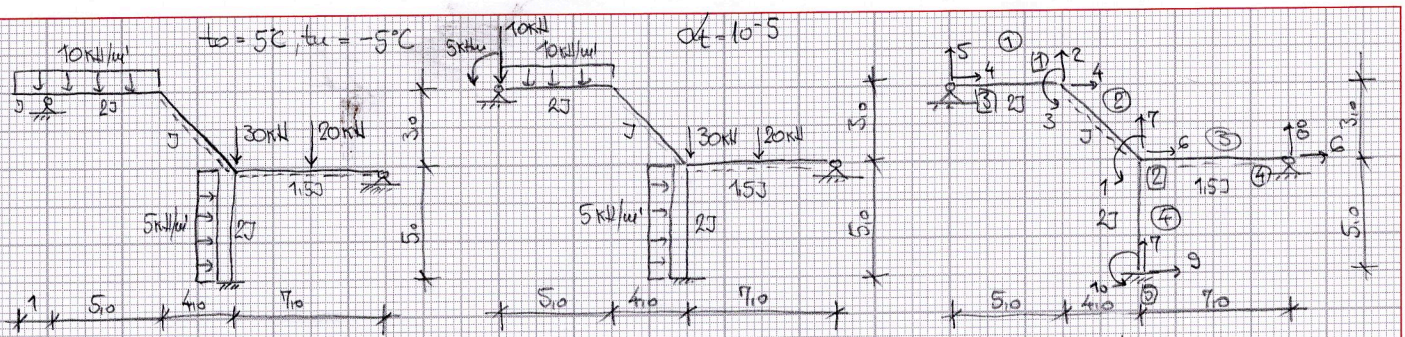


$t_0 = 5°C$, $t_u = -5°C$ $\quad \alpha_t = 10^{-5}$

Štap	i	k	λ	μ	ℓ	EJ	EF
1	3	1	1	0	5,00	27000	
2	1	2	0,80	−0,60	5,00	13500	1800000
3	2	4	1	0	7,00	20250	
4	2	5	0	−1	5,00	27000	

$E = 3 \cdot 10^7\ kN/m^2$; $b/h_c = 20/30\ cm$

$J = J_c = 4,5 \cdot 10^{-4}\ m^4$ $\quad F = F_c = 6 \cdot 10^{-2}\ m^2$

$1,5J \rightarrow h = 34,34\ cm \rightarrow J = 6,75 \cdot 10^{-4}\ m^4$

$F = 6,868 \cdot 10^{-2}\ m^2$

$2J \rightarrow h = 37,80\ cm \rightarrow J = 9 \cdot 10^{-4}\ m^4$

$F = 11,96 \cdot 10^{-2}\ m^2$

- Štap 1

$$K_1^* = K_1 = \begin{bmatrix} \frac{3EJ}{\ell^3} & -\frac{3EJ}{\ell^3} & \frac{3EJ}{\ell^2} \\ & \frac{3EJ}{\ell^3} & -\frac{3EJ}{\ell^2} \\ \text{„S"} & & \frac{3EJ}{\ell} \end{bmatrix} = \begin{matrix} 5 & 2 & 3 \\ \begin{bmatrix} 648 & -648 & 3240 \\ & 648 & -3240 \\ \text{„S"} & & 16200 \end{bmatrix} & \begin{matrix} 5 \\ 2 \\ 3 \end{matrix} \end{matrix}$$

$$K_2 = \begin{bmatrix} \frac{EF}{\ell} & 0 & 0 & -\frac{EF}{\ell} & 0 & \\ & \frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} & 0 & -\frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} \\ & & \frac{4EJ}{\ell} & 0 & -\frac{6EJ}{\ell^2} & \frac{2EJ}{\ell} \\ & & & \frac{EF}{\ell} & 0 & 0 \\ & & & & \frac{12EJ}{\ell^3} & -\frac{6EJ}{\ell^2} \\ \text{„S"} & & & & & \frac{4EJ}{\ell} \end{bmatrix} =$$

$$K_2 = \begin{matrix} 4 & 2 & 3 & 6 & 7 & 1 \\ \begin{bmatrix} 360000 & 0 & 0 & -360000 & 0 & 0 \\ 0 & 1296 & 3240 & 0 & -1296 & 3240 \\ 0 & 3240 & 10800 & 0 & -3240 & 5400 \\ -360000 & 0 & 0 & 360000 & 0 & 0 \\ 0 & -1296 & -3240 & 0 & 1296 & -3240 \\ 0 & 3240 & 5400 & 0 & -3240 & 10800 \end{bmatrix} & \begin{matrix} 4 \\ 2 \\ 3 \\ 6 \\ 7 \\ 1 \end{matrix} \end{matrix} \quad T_2 = \begin{bmatrix} 0,80 & -0,60 & & & & \\ 0,60 & 0,80 & & & & \\ & & 1 & & & \\ & & & 0,80 & -0,60 & \\ & & & 0,60 & 0,80 & \\ & & & & & 1 \end{bmatrix}$$

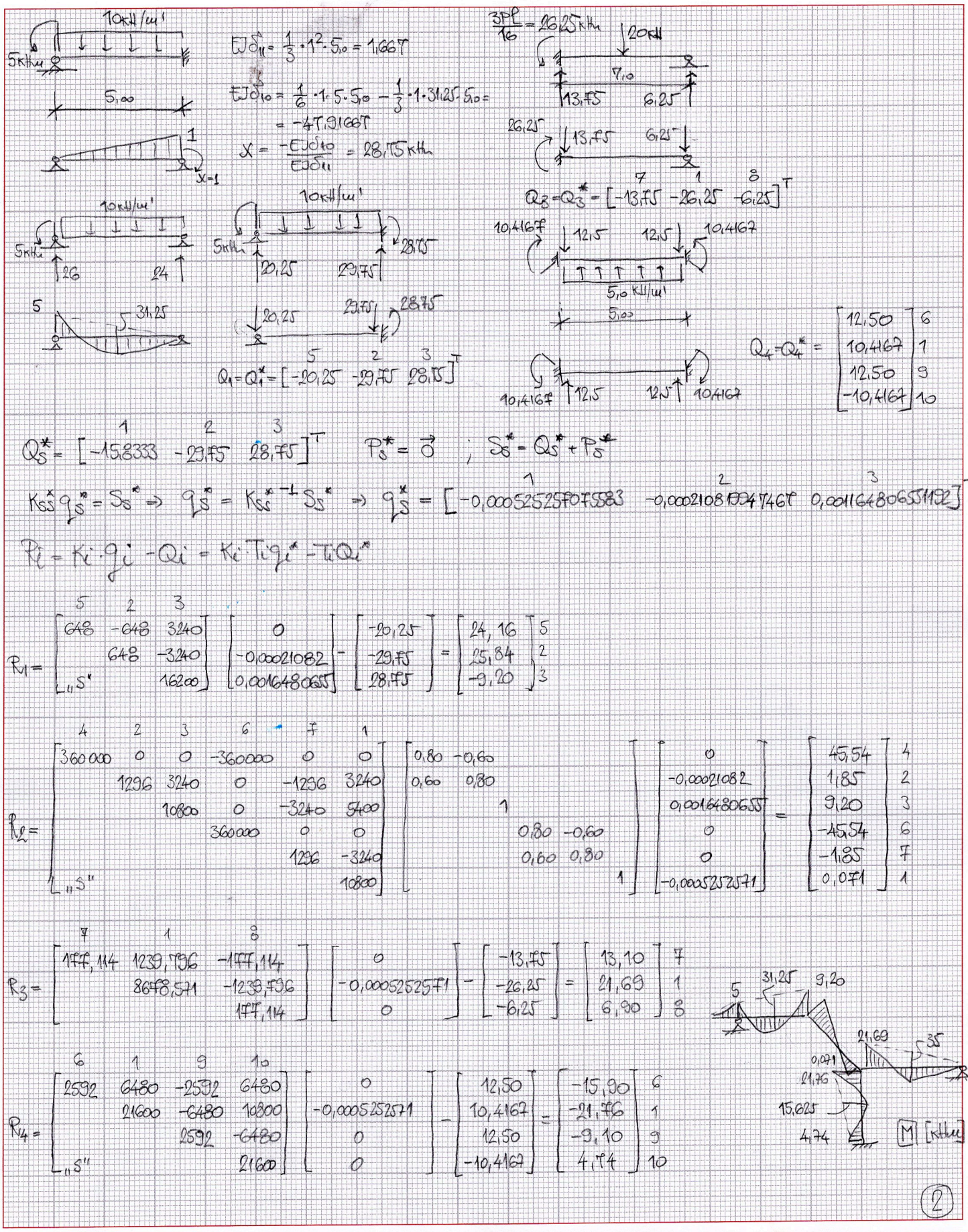
$$K_2^* = T_2 K_2 T_2 = \begin{matrix} 4 & 2 & 3 & 6 & 7 & 1 \\ \begin{bmatrix} 230866,56 & -172177,92 & 1944,0 & -230866,56 & 172177,92 & 1944,0 \\ & 130429,44 & 2592,0 & 172177,92 & -130429,44 & 2592,0 \\ & & 10800 & -1944,0 & -2592,0 & 5400 \\ & & & 230866,56 & -172177,92 & -1944,0 \\ & & & & 130429,44 & -2592,0 \\ \text{„S"} & & & & & 10800 \end{bmatrix} & \begin{matrix} 4 \\ 2 \\ 3 \\ 6 \\ 7 \\ 1 \end{matrix} \end{matrix}$$

$$K_4 = K_4^* = \begin{matrix} 6 & 1 & 9 & 10 \\ \begin{bmatrix} 2592 & 6480 & -2592 & 6480 \\ & 21600 & -6480 & 10800 \\ & & 2592 & -6480 \\ \text{„S"} & & & 21600 \end{bmatrix} & \begin{matrix} 6 \\ 1 \\ 9 \\ 10 \end{matrix} \end{matrix}$$

$$K_3^* = K_3 = \begin{bmatrix} \frac{3EJ}{\ell^3} & \frac{3EJ}{\ell^2} & -\frac{3EJ}{\ell^3} \\ & \frac{3EJ}{\ell} & -\frac{3EJ}{\ell^2} \\ \text{„S"} & & \frac{3EJ}{\ell^3} \end{bmatrix} = \begin{matrix} 7 & 1 & 8 \\ \begin{bmatrix} 177,114 & 1239,796 & -177,114 \\ & 8678,571 & -1239,796 \\ \text{„S"} & & 177,114 \end{bmatrix} & \begin{matrix} 7 \\ 1 \\ 8 \end{matrix} \end{matrix}$$

$$K_{SS}^* = \begin{matrix} 1 & 2 & 3 \\ \begin{bmatrix} 410778,571 & 2592 & 5400 \\ & 131077,44 & -648 \\ \text{„S"} & & 27000 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix}$$

(1)



$$EJ\delta_{11} = \frac{1}{3}\cdot 1^2 \cdot 5{,}0 = 1{,}667$$

$$EJ\delta_{10} = \frac{1}{6}\cdot 1\cdot 5\cdot 5{,}0 - \frac{1}{3}\cdot 1\cdot 31{,}25\cdot 5{,}0 = -47,9167$$

$$X = \frac{-EJ\delta_{10}}{EJ\delta_{11}} = 28{,}75\ \text{kNm}$$

$$Q_1 = Q_1^* = [-20{,}25 \quad -29{,}75 \quad 28{,}75]^T \quad (5,\ 2,\ 3)$$

$$Q_3 = Q_3^* = [-13{,}75 \quad -26{,}25 \quad -6{,}25]^T \quad (7,\ 1,\ 8)$$

$$Q_4 = Q_4^* = \begin{bmatrix} 12{,}50 \\ 10{,}4167 \\ 12{,}50 \\ -10{,}4167 \end{bmatrix} \begin{matrix} 6 \\ 1 \\ 9 \\ 10 \end{matrix}$$

$$Q_S^* = [-15{,}8333 \quad -29{,}75 \quad 28{,}75]^T \quad (1,\ 2,\ 3) \qquad P_S^* = \vec{0} \ ; \quad S_S^* = Q_S^* + P_S^*$$

$$K_{SS}^* \cdot q_S^* = S_S^* \Rightarrow q_S^* = K_{SS}^{*-1} S_S^* \Rightarrow q_S^* = [-0{,}000525250707583 \quad -0{,}000210810247467 \quad 0{,}00116480655192]^T \quad (1,\ 2,\ 3)$$

$$R_i = K_i \cdot q_i - Q_i = K_i T_i q_i^* - T_i Q_i^*$$

$$R_1 = \begin{bmatrix} 648 & -648 & 3240 \\ & 648 & -3240 \\ \text{„}S\text{“} & & 16200 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}00021082 \\ 0{,}00164806 55 \end{bmatrix} - \begin{bmatrix} -20{,}25 \\ -29{,}75 \\ 28{,}75 \end{bmatrix} = \begin{bmatrix} 24{,}16 \\ 25{,}84 \\ -3{,}20 \end{bmatrix} \begin{matrix} 5 \\ 2 \\ 3 \end{matrix} \quad (5,\ 2,\ 3)$$

$$R_2 = \begin{bmatrix} 360000 & 0 & 0 & -360000 & 0 & 0 \\ & 1296 & 3240 & 0 & -1296 & 3240 \\ & & 10800 & 0 & -3240 & 5400 \\ & & & 360000 & 0 & 0 \\ & & & & 1296 & -3240 \\ \text{„}S\text{“} & & & & & 10800 \end{bmatrix} \begin{bmatrix} 0{,}80 & -0{,}60 & & & & \\ 0{,}60 & 0{,}80 & & & & \\ & & 1 & & & \\ & & & 0{,}80 & -0{,}60 & \\ & & & 0{,}60 & 0{,}80 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}00021082 \\ 0{,}00164806 55 \\ 0 \\ 0 \\ -0{,}000525250 71 \end{bmatrix} = \begin{bmatrix} 45{,}54 \\ 1{,}85 \\ 9{,}20 \\ -45{,}54 \\ -1{,}85 \\ 0{,}071 \end{bmatrix} \begin{matrix} 4 \\ 2 \\ 3 \\ 6 \\ 7 \\ 1 \end{matrix} \quad (4,\ 2,\ 3,\ 6,\ 7,\ 1)$$

$$R_3 = \begin{bmatrix} 177{,}114 & 1239{,}796 & -177{,}114 \\ & 8648{,}541 & -1239{,}796 \\ & & 177{,}114 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}000525250 71 \\ 0 \end{bmatrix} - \begin{bmatrix} -13{,}75 \\ -26{,}25 \\ -6{,}25 \end{bmatrix} = \begin{bmatrix} 13{,}10 \\ 21{,}69 \\ 6{,}90 \end{bmatrix} \begin{matrix} 7 \\ 1 \\ 8 \end{matrix} \quad (7,\ 1,\ 8)$$

$$R_4 = \begin{bmatrix} 2592 & 6480 & -2592 & 6480 \\ & 21600 & -6480 & 10800 \\ & & 2592 & -6480 \\ \text{„}S\text{“} & & & 21600 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}000525250 71 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 12{,}50 \\ 10{,}4167 \\ 12{,}50 \\ -10{,}4167 \end{bmatrix} = \begin{bmatrix} -15{,}90 \\ -21{,}76 \\ -3{,}10 \\ 4{,}74 \end{bmatrix} \begin{matrix} 6 \\ 1 \\ 9 \\ 10 \end{matrix} \quad (6,\ 1,\ 9,\ 10)$$

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Utjecaj temperaturne razlike $t_0 = 5°C$ $t_u = -5°C$; $t = 0$, $\Delta t = 10°C$

$$Q_2^T = EJ\alpha\frac{\Delta t}{h}[0\ 1\ 0\ -1] = 13500 \cdot 10^{-5} \cdot \frac{10}{0{,}30}[0\ 1\ 0\ -1] = [0\ 4{,}50\ 0\ -4{,}50]$$

$$Q_2^{*T} = T_2^T Q_2 = [0\ 0\ 4{,}50\ 0\ 0\ -4{,}50] \quad (4\ 2\ 3\ 6\ 7\ 1)$$

$$Q_3 = Q_3^* = 1{,}50\,EJ\alpha\frac{\Delta t}{h}\left[\frac{1}{e}\ 1\ -\frac{1}{e}\right] = 1{,}50 \cdot 20250 \cdot 10^{-5} \cdot \frac{10}{0{,}3434}\left[\frac{1}{7{,}10}\ 1\ -\frac{1}{7{,}10}\right] = [1{,}264\ \ 8{,}845\ \ -1{,}264] \quad (7\ 1\ 8)$$

$$Q_5^* = [4{,}345\ \ 0\ \ 4{,}50] \quad (1\ 2\ 3)$$

$$K_{55}^* q_5^* = Q_5^* \Rightarrow q_5^* = K_{55}^{*-1} Q_5^* = [0{,}086193795140597\ \ -0{,}000365841670575\ \ 0{,}149404274384553] \cdot 10^{-3} \quad (1\ 2\ 3)$$

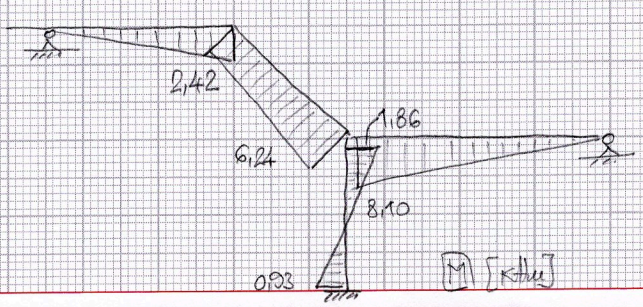
$$R_i = K_i q_i - Q_i = K_i T_i q_i^* - T_i Q_i^*$$

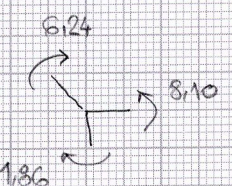
$$R_1 = \begin{bmatrix} 648 & -648 & 3240 \\ & 648 & -3240 \\ \text{„S"} & & 16200 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}000365842 \\ 0{,}149404274 \end{bmatrix} = \begin{bmatrix} 0{,}48 \\ -0{,}48 \\ 2{,}42 \end{bmatrix} \begin{matrix} 5 \\ 2 \\ 3 \end{matrix} \quad (5\ 2\ 3)$$

$$R_2 = \begin{bmatrix} 360000 & 0 & 0 & -360000 & 0 & 0 \\ 0 & 1296 & 3240 & 0 & -1296 & 3240 \\ 0 & 3240 & 10800 & 0 & -3240 & 5400 \\ -360000 & 0 & 0 & 360000 & 0 & 0 \\ 0 & -1296 & -3240 & 0 & 1296 & -3240 \\ 0 & 3240 & 5400 & 0 & -3240 & 10800 \end{bmatrix} \begin{bmatrix} 0{,}80 & -0{,}60 & & & & \\ 0{,}60 & 0{,}80 & & & & \\ & & 1 & & & \\ & & & 0{,}80 & -0{,}60 & \\ & & & 0{,}60 & 0{,}80 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}000365842 \\ 0{,}149404274 \\ 0 \\ 0 \\ 0{,}086193795 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 4{,}50 \\ 0 \\ 0 \\ -4{,}50 \end{bmatrix} = \begin{bmatrix} 0{,}21 \\ 0{,}76 \\ -2{,}42 \\ -0{,}21 \\ -0{,}76 \\ 6{,}24 \end{bmatrix} \begin{matrix} 4 \\ 2 \\ 3 \\ 6 \\ 4 \\ 1 \end{matrix} \quad (4\ 2\ 3\ 6\ 4\ 1)$$

$$R_3 = \begin{bmatrix} 177{,}114 & 1239{,}796 & -177{,}114 \\ & 8678{,}571 & -1239{,}796 \\ \text{„S"} & & 177{,}114 \end{bmatrix} \begin{bmatrix} 0 \\ 0{,}086193795951 \\ 0 \end{bmatrix} - \begin{bmatrix} 1{,}264 \\ 8{,}845 \\ -1{,}264 \end{bmatrix} = \begin{bmatrix} -1{,}16 \\ -8{,}10 \\ 1{,}16 \end{bmatrix} \begin{matrix} 7 \\ 1 \\ 8 \end{matrix} \quad (7\ 1\ 8)$$

$$R_4 = \begin{bmatrix} 2592 & 6480 & -2592 & 6480 \\ & 21600 & -6480 & 10800 \\ & & 2592 & -6480 \\ & & & 21600 \end{bmatrix} \begin{bmatrix} 0 \\ 0{,}086193795951 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0{,}56 \\ 1{,}86 \\ -0{,}56 \\ 0{,}93 \end{bmatrix} \begin{matrix} 6 \\ 1 \\ 9 \\ 10 \end{matrix} \quad (6\ 1\ 9\ 10)$$





3

= 2.074074

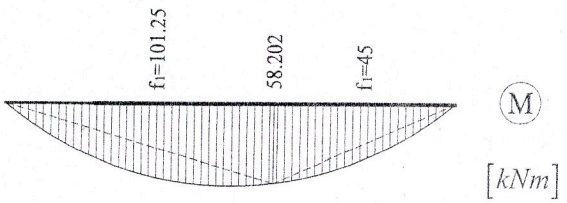
ds = –400

71 1
071 2
4
29 5

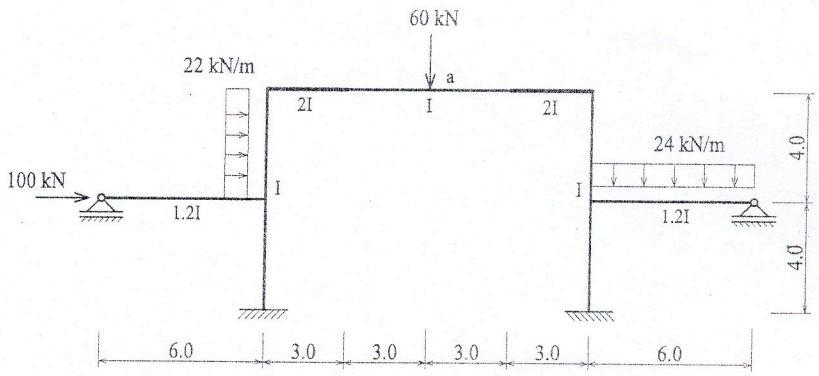
2.725
58.202
7.275

-štap 1:

$$\mathbf{q}_1^* = \frac{1}{EI}\begin{bmatrix} 0 \\ 455.5224 \\ -241.9294 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 4 \\ 1 \\ 2 \\ 4 \\ 5 \end{matrix} \quad \rightarrow \quad \mathbf{R}_1 = \mathbf{K}_1\mathbf{q}_1 - \mathbf{Q}_1 = \mathbf{K}_1\mathbf{T}_1\mathbf{q}_1^* - \mathbf{Q}_1 = \begin{bmatrix} 64.859 \\ 58.212 \\ -55.141 \end{bmatrix}$$

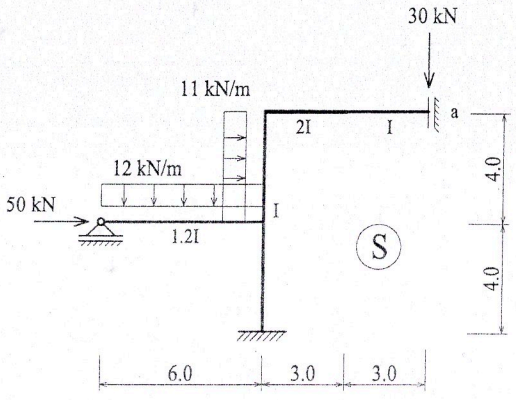


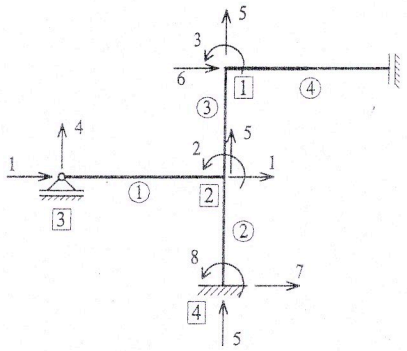
5. Za dati nosač:
a) Za simetričan slučaj opterećenja nacrtati dijagram momenata
b) Odrediti vertikalno pomeranje tačke **a**
Napomena: Uticaj normalnih sila na deformaciju zanemariti.



Rešenje:

a) Simetrija:





Nepoznata pomeranja su q_1^*, q_2^*, q_3^*,

Matrice krutosti štapova

-štap 1:

$$\mathbf{K}_1^* = \mathbf{K}_1 = EI \begin{array}{c} \\ 4 \\ 5 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 4 & 5 & 2 \end{array} \\ \begin{bmatrix} 0.0167 & -0.0167 & 0.100 \\ -0.0167 & 0.0167 & -0.100 \\ 0.100 & -0.100 & 0.600 \end{bmatrix} \end{array} \begin{array}{c} \\ 4 \\ 5 \\ 2 \end{array}$$

-štap 2:

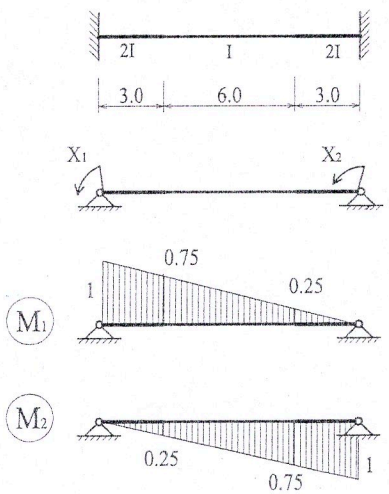
$$\mathbf{K}_2^* = \mathbf{K}_2 = EI \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.0 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array}$$

-štap 3:

$$\mathbf{K}_3^* = \mathbf{K}_3 = EI \begin{array}{c} \begin{matrix} 6 & 3 & 1 & 2 \end{matrix} \\ \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.0 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.0 \end{bmatrix} \end{array} \begin{matrix} 6 \\ 3 \\ 1 \\ 2 \end{matrix}$$

-štap 4:

Štap 4 je štap tipa "s". Štap tipa "s" nastaje kada osa simetrije polovi horizontalni štap tipa "k" isključivo u simetriji. Kako je štap 4 i promenljivog poprečnog preseka, matrica krutosti mora se izvesti iz izvornog štapa tipa "k":



$$I_c = I$$

$$EI_c\alpha_{ii'} = \frac{I_c}{I}\int_s M_1^2 ds = 2.8125$$

$$EI_c\beta_{ii'} = \frac{I_c}{I}\int_s M_1 M_2 ds = -1.6875$$

$$\Delta = \alpha_{ii'}^2 - \beta_{ii'}^2 = 2.1826^2 - 1.6875^2 = 5.0625$$

$$a_{ii'} = \frac{\alpha_{ii'}}{\Delta} = 0.5556$$

$$b_{ii'} = -\frac{\beta_{ii'}}{\Delta} = 0.3333$$

$$e_{is} = \alpha_{ii'} - b_{ii'} = 0.2222$$

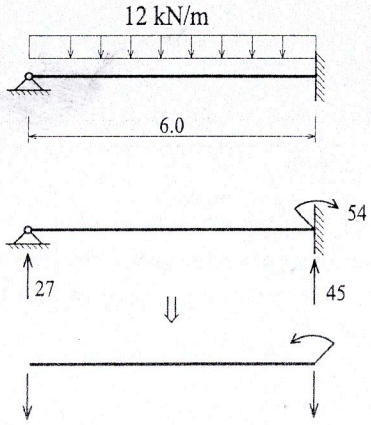
$$\mathbf{K}_4 = \mathbf{K}_4^* = EI \begin{array}{c} \begin{matrix} 5 & 3 \end{matrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0.2222 \end{bmatrix} \end{array} \begin{matrix} 5 \\ 3 \end{matrix}$$

-submatrica krutosti nepoznatih:

$$\mathbf{K}_{nn}^* = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0.375 & 0 & -0.375 \\ 0 & 2.60 & 0.50 \\ -0.375 & 0.50 & 1.2222 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

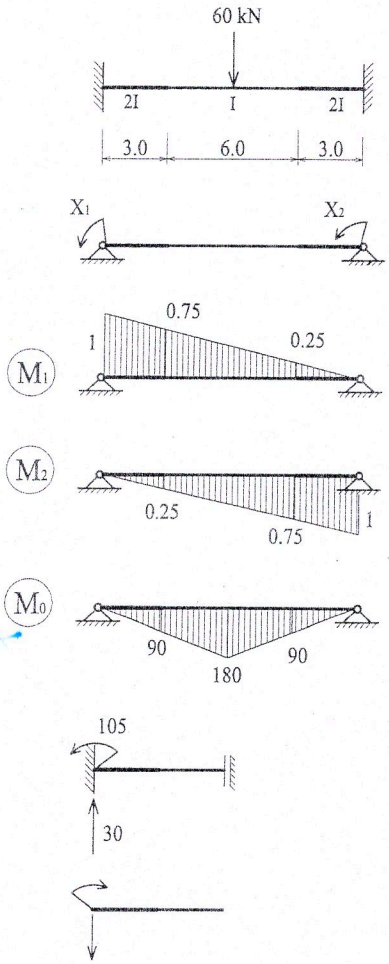
Vektor ekvivalentnog opterećenja

-štap 1:



$$\mathbf{Q}_1^* = \begin{bmatrix} -27 \\ -45 \\ 54 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 2 \end{matrix}$$

-štap 4:



$$EI\delta_{11} = \frac{I_c}{I}\int_s M_1^2 ds = 2.8125$$

$$EI\delta_{12} = \frac{I_c}{I}\int_s M_1 M_2 ds = -1.6875$$

$$EI\delta_{22} = \frac{I_c}{I}\int_s M_2^2 ds = 2.8125$$

$$EI_c\delta_{10} = \frac{I_c}{I}\int_s M_1 M_0 ds = -472.50$$

$$EI_c\delta_{20} = \frac{I_c}{I}\int_s M_2 M_0 ds = 472.5$$

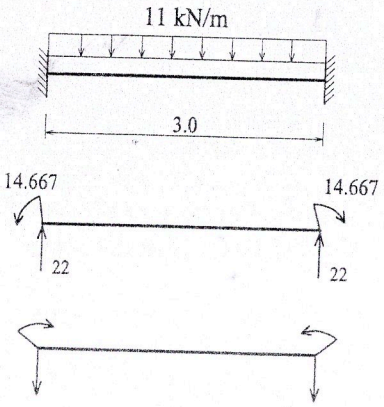
$$\mathbf{DX} + \boldsymbol{\delta} = \mathbf{0}$$

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} \delta_{10} \\ \delta_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = 105 \qquad x_2 = -105$$

$$\mathbf{Q}_4 = \mathbf{Q}_4^* = \begin{bmatrix} -30 \\ -105 \end{bmatrix} \begin{matrix} 5 \\ 3 \end{matrix}$$

-štap 3:



$$\mathbf{Q}_3^* = \begin{bmatrix} -22 \\ 14.667 \\ -22 \\ -14.667 \end{bmatrix} \begin{matrix} 6 \\ 3 \\ 1 \\ 2 \end{matrix}$$

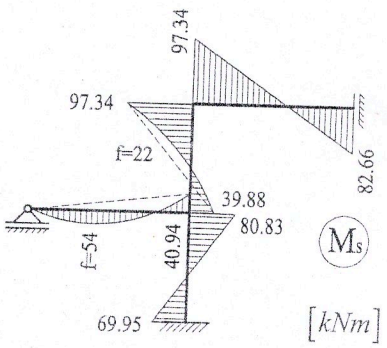
Vektor slobodnih članova

$$\mathbf{Q}_n^* = \begin{bmatrix} 22.0 \\ 39.333 \\ -90.333 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \qquad \mathbf{P}_n^* = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \qquad \rightarrow \qquad \mathbf{S}_n^* = \mathbf{Q}_n^* + \mathbf{P}_n^* = \begin{bmatrix} 72.0 \\ 39.333 \\ -90.333 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

-uslovne jednačine i rešenje:

$$\mathbf{K}_{nn}^* \mathbf{q}_n^* = \mathbf{S}_{nn}^* \qquad \rightarrow \qquad \mathbf{q}_n^* = \frac{1}{EI} \begin{bmatrix} 157.519 \\ 21.759 \\ -34.481 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Sile na krajevima štapa: $\mathbf{R}_i = \mathbf{K}_i \mathbf{q}_i - \mathbf{Q}_i \rightarrow$ M_s

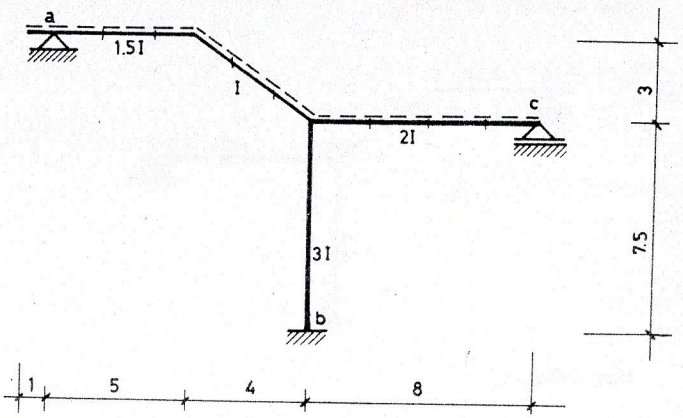


MAJ 1985.

ZADACI

1.

Za dati nosač, primenom približne metode deformacije, nacrtati uticajnu liniju za vertikalnu reakciju oslonca a, sa ordinatama u označenim tačkama.

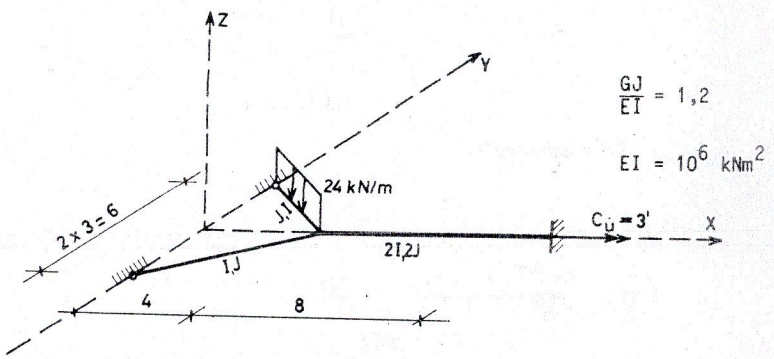


2.

Za dati roštilj formirati matricu krutosti sistema i vektor ekvivalentnog opterećenja. Sračunati pomeranja čvorova i sile u štapovima usled:

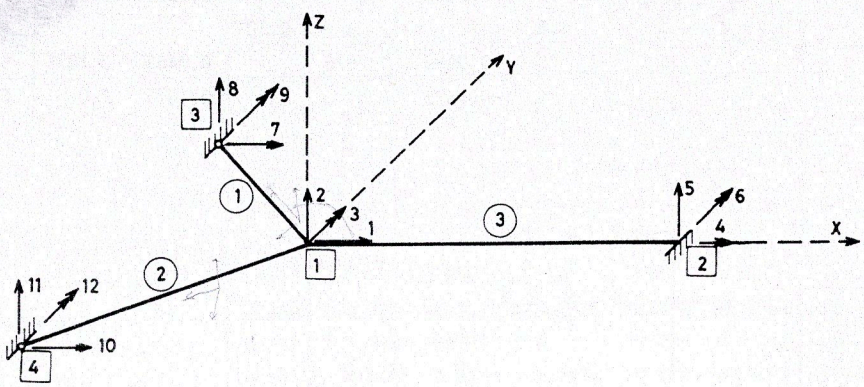
a) opterećenja

b) obrtanja ukliještenja $c_u = 3$



2.

Šema nosača



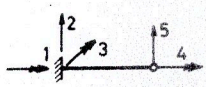
Nepoznata pomeranja: 1,2,3

	i	k	cos	sin	J	I	l
1	1	3	-0,8	0,6	J	I	5
2	1	4	-0,8	-0,6	J	I	5
3	1	2	1	0	2J	2I	8

$\frac{GJ}{EI} = 1,2$

Matrice krutosti štapova

$$K_g = EI \begin{bmatrix} & 1 & 2 & 3 & 4 & 5 & \\ & GJ/EI\cdot l & & & GJ/EI\cdot l & & 1 \\ & & 3/l^3 & -3/l^2 & & -3/l^2 & 2 \\ & & -3/l^2 & 3/l & & 3/l^2 & 3 \\ & -GJ/EI\cdot l & & & GJ/EI\cdot l & & 4 \\ & & -3/l^3 & 3/l^2 & & 3/l^3 & 5 \end{bmatrix}$$



$$K_1 = K_2 = EI \begin{bmatrix} 0,24 & & & -0,24 & \\ & 0,024 & -0,12 & & -0,024 \\ & -0,12 & 0,6 & & 0,12 \\ -0,24 & & & 0,24 & \\ & -0,024 & 0,12 & & 0,024 \end{bmatrix}$$

$$\mathbf{K}_3 = \mathbf{K}_3^* = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0,30 & & & -0,30 & & \\ & 0,0469 & -0,1875 & & -0,0469 & -0,1875 \\ & -0,1875 & 1,0 & & 0,1875 & 0,5 \\ -0,30 & & & 0,30 & & \\ & -0,0469 & 0,1875 & & 0,0469 & 0,1875 \\ & -0,1875 & 0,5 & & 0,1875 & 1,0 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Matrice transformacije

$$\mathbf{T}_1 = \begin{bmatrix} -0,8 & 0 & 0,6 & & & \\ 0 & 1 & 0 & & & \\ -0,6 & 0 & -0,8 & & & \\ & & & -0,8 & 0 & 0,6 \\ & & & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{T}_2 = \begin{bmatrix} -0,8 & 0 & -0,6 & & & \\ 0 & 1 & 0 & & & \\ 0,6 & 0 & -0,8 & & & \\ & & & -0,8 & 0 & -0,6 \\ & & & 0 & 1 & 0 \end{bmatrix}$$

$$\hat{\mathbf{K}}_i = \mathbf{K}_i \mathbf{T}_i$$

$$\hat{\mathbf{K}}_1 = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \begin{bmatrix} -0,192 & 0 & 0,144 & 0,192 & 0 & -0,144 \\ 0,072 & 0,024 & 0,096 & 0 & -0,024 & 0 \\ -0,36 & -0,12 & -0,48 & 0 & 0,12 & 0 \\ 0,192 & 0 & -0,144 & -0,192 & 0 & 0,144 \\ -0,072 & -0,024 & -0,096 & 0 & 0,024 & 0 \end{bmatrix} \end{array}$$

$$\hat{\mathbf{K}}_2 = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 10 & 11 & 12 \end{matrix} \\ \begin{bmatrix} -0,192 & 0 & -0,144 & 0,192 & 0 & 0,144 \\ -0,072 & 0,024 & 0,096 & 0 & -0,024 & 0 \\ 0,36 & -0,12 & -0,48 & 0 & 0,12 & 0 \\ 0,192 & 0 & 0,144 & -0,192 & 0 & -0,144 \\ 0,072 & -0,024 & -0,096 & 0 & 0,024 & 0 \end{bmatrix} \end{array}$$

$\mathbf{K}_1^* =$

$\mathbf{K}_1^* =$

$\mathbf{K}_2^* =$

Matr

$$\mathbf{K}^* = EI \begin{array}{c} \begin{matrix} 1 & 2 \end{matrix} \\ \begin{bmatrix} 1,0392 & 0 \\ 0 & 0,094 \\ 0 & 0,004 \\ -0,3 & \\ & -0,046 \\ & -0,187 \\ -0,1536 & \\ -0,072 & -0,024 \\ 0,1152 & \\ -0,1536 & \\ 0,072 & -0,024 \\ -0,1152 & \end{bmatrix} \end{array}$$

$$K_i^* = T_i^T \hat{K}_i$$

$$K_1^* = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 7 & 8 & 9 \end{matrix} \\ \begin{bmatrix} 0,3696 & 0,072 & 0,1728 & -0,1536 & -0,072 & 0,1152 \\ 0,072 & 0,024 & 0,096 & 0 & -0,024 & 0 \\ 0,1728 & 0,096 & 0,4704 & 0,1152 & -0,096 & -0,0864 \\ -0,1536 & 0 & 0,1152 & 0,1536 & 0 & -0,1152 \\ -0,072 & -0,024 & -0,096 & 0 & 0,024 & 0 \\ 0,1152 & 0,0 & -0,0864 & -0,1152 & 0 & 0,0864 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{matrix}$$

$$K_2^* = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 10 & 11 & 12 \end{matrix} \\ \begin{bmatrix} 0,3696 & -0,072 & -0,1728 & -0,1536 & 0,072 & -0,1152 \\ -0,072 & 0,024 & 0,096 & 0 & -0,024 & 0 \\ -0,1728 & 0,096 & 0,4704 & -0,1152 & -0,096 & -0,0864 \\ -0,1536 & 0 & -0,1152 & 0,1536 & 0 & 0,1152 \\ 0,072 & -0,024 & -0,096 & 0 & 0,024 & 0 \\ -0,1152 & 0 & -0,0864 & 0,1152 & 0 & 0,0864 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 10 \\ 11 \\ 12 \end{matrix}$$

Matrica krutosti sistema

$$K^* = EI \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{matrix} \\ \left[\begin{array}{cccccccccccc} 1,0392 & 0 & 0 & -0,3 & & & -0,1536 & -0,072 & 0,1152 & -0,1536 & 0,072 & -0,1152 \\ 0 & 0,0949 & 0,0045 & & -0,0469 & -0,1875 & & -0,024 & & & -0,024 & \\ 0 & 0,0045 & 1,9408 & & -0,1875 & 0,5 & 0,1152 & -0,096 & -0,0864 & -0,1152 & -0,096 & -0,0864 \\ -0,3 & & & 0,3 & & & & & & & & \\ & -0,0469 & 0,1875 & & 0,0469 & 0,1875 & & & & & & \\ & -0,1875 & 0,5 & & 0,1875 & 1,0 & & & & & & \\ -0,1536 & & 0,1152 & & & & 0,1536 & & -0,1152 & & & \\ -0,072 & -0,024 & -0,096 & & & & & 0,024 & & & & \\ 0,1152 & & -0,0864 & & & & -0,1152 & & 0,0864 & & & \\ -0,1536 & & -0,1152 & & & & & & & 0,1536 & & 0,1152 \\ 0,072 & -0,024 & -0,096 & & & & & & & & 0,024 & \\ -0,1152 & & -0,0864 & & & & & & & 0,1152 & & 0,0864 \end{array}\right] \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix}$$

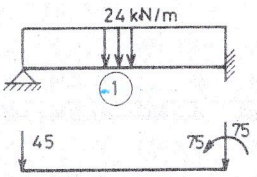
$$\begin{matrix} & 6 \\ & \left[\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right. \\ 469 & -0,1875 \\ 875 & 0,5 \\ & \\ 469 & 0,1875 \\ 875 & 1,0 \end{matrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$\begin{matrix} 9 \\ \left[\begin{array}{c} -0,144 \\ 0 \\ 0 \\ 0,144 \\ 0 \end{array}\right. \end{matrix}$$

$$\begin{matrix} 12 \\ \left[\begin{array}{c} 0,144 \\ 0 \\ 0 \\ -0,144 \\ 0 \end{array}\right. \end{matrix}$$

a) Opterećenje

Vektor ekvivalentnog opterećenja



$$Q_1 = \begin{bmatrix} 0 \\ -75 \\ 75 \\ 0 \\ -45 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \end{matrix} \qquad Q_1^* = T_1^T Q_1 = \begin{bmatrix} -45 \\ -75 \\ -60 \\ 0 \\ -45 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{matrix} = \begin{bmatrix} Q_n^* \\ Q_p^* \end{bmatrix}$$

Uslovne jednačine i rešenje

$$K_{nn}^* \, q_n^* = Q_n^* \qquad q_n^* = \frac{1}{EI} \begin{bmatrix} -43{,}3025 \\ -788{,}9264 \\ -29{,}0859 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Vektori sila na krajevima štapova

$$R_i = \hat{K}_i \, q_i^* - Q_i$$

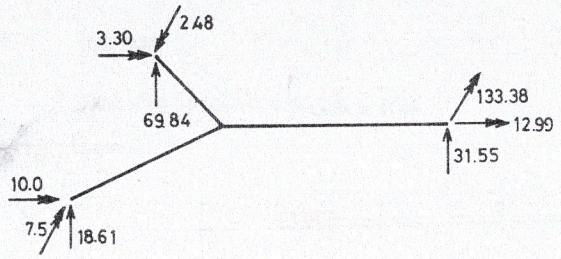
$$R_1 = \begin{bmatrix} 4{,}126 \\ -24{,}844 \\ 124{,}221 \\ -4{,}126 \\ 24{,}844 \end{bmatrix} - \begin{bmatrix} 0 \\ -75 \\ 75 \\ 0 \\ -45 \end{bmatrix} = \begin{bmatrix} 4{,}126 \\ 50{,}156 \\ 49{,}221 \\ -4{,}126 \\ 69{,}844 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 12{,}503 \\ -18{,}609 \\ 93{,}044 \\ -12{,}503 \\ 18{,}609 \end{bmatrix} \qquad R_3 = \begin{bmatrix} -12{,}991 \\ -31{,}547 \\ 118{,}838 \\ 12{,}991 \\ 31{,}547 \\ 133{,}381 \end{bmatrix}$$

Vektor reakcija oslonaca

$$S_p^* = K_{pn}^* q_n^* - Q_p^* = \begin{bmatrix} 12{,}991 \\ 31{,}547 \\ 133{,}381 \\ 3{,}301 \\ 24{,}844 \\ -2{,}475 \\ 10{,}002 \\ 18{,}609 \\ 7{,}502 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -45 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12{,}991 \\ 31{,}547 \\ 133{,}381 \\ 3{,}301 \\ 69{,}844 \\ -2{,}475 \\ 10{,}002 \\ 18{,}609 \\ 7{,}502 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix}$$

$$T_1^T Q_1 = \begin{bmatrix} -45 \\ -75 \\ -60 \\ \hline 0 \\ -45 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ \\ 7 \\ 8 \\ 9 \end{matrix} = \begin{bmatrix} Q_n^* \\ Q_p^* \end{bmatrix}$$



$\Sigma Z = 0$

$\Sigma M_t = 0$

$\Sigma M_y = 0$

b) Obrtanje uklještenja

$q_4^* = 3' = 0{,}0008726$ rad

$K_{nn}\, q_n^* + K_{np}\, q_p^* = 0$

$K_{nn}\, q_n^* = - K_{np}\, q_4^*$

Uslovne jednačine i rešenja

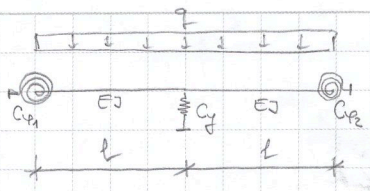
$$\begin{bmatrix} 1{,}0392 & & \\ & 0{,}0949 & 0{,}0045 \\ & 0{,}0045 & 1{,}9408 \end{bmatrix} \begin{bmatrix} q_1^* \\ q_2^* \\ q_3^* \end{bmatrix} = \begin{bmatrix} 2{,}618 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \; 2 \cdot 10^{-4}\, EI$$

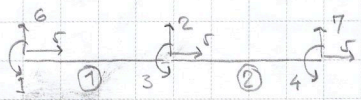
$$q_n^* = 10^{-4} \begin{bmatrix} 2{,}5192 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Vektori sila na krajevima štapova: $R_i = \hat{K}_i\, q_i^*$

$$R_1 = \begin{bmatrix} -\,48{,}4 \\ 18{,}1 \\ -\,90{,}7 \\ 48{,}4 \\ -\,18{,}1 \end{bmatrix} \qquad R_2 = \begin{bmatrix} -\,48{,}4 \\ -\,18{,}1 \\ 90{,}7 \\ 48{,}4 \\ 18{,}1 \end{bmatrix}$$

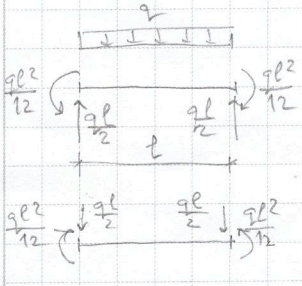
$$R_3 = \begin{bmatrix} -\,186{,}2 \\ 0 \\ 0 \\ 186{,}2 \\ 0 \\ 0 \end{bmatrix}$$





$$K_1 = \frac{EJ}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ \text{"S"} & & & 4l^2 \end{bmatrix}\begin{matrix}6\\1\\2\\3\end{matrix} \qquad K_2 = \frac{EJ}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ \text{"S"} & & & 4l^2 \end{bmatrix}\begin{matrix}2\\3\\7\\4\end{matrix}$$

$$K_{SS}^{*} = \begin{bmatrix} \frac{4EJ}{l}+C_{\varphi 1} & -\frac{6EJ}{l^2} & \frac{2EJ}{l} & 0 \\ & \frac{24EJ}{l^3}+C_y & 0 & \frac{6EJ}{l^2} \\ & & \frac{8EJ}{l} & \frac{2EJ}{l} \\ \text{"S"} & & & \frac{4EJ}{l}+C_{\varphi 2} \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix}$$



$$Q_1 = \begin{bmatrix} -\frac{ql}{2} \\ -\frac{ql^2}{12} \\ -\frac{ql}{2} \\ \frac{ql^2}{12} \end{bmatrix}\begin{matrix}6\\1\\2\\3\end{matrix} \qquad Q_2 = \begin{bmatrix} -\frac{ql}{2} \\ -\frac{ql^2}{12} \\ -\frac{ql}{2} \\ \frac{ql^2}{12} \end{bmatrix}\begin{matrix}2\\3\\7\\4\end{matrix}$$

$$Q_S^{*} = \begin{bmatrix} -\frac{ql^2}{12} \\ -ql \\ 0 \\ \frac{ql^2}{12} \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix} \qquad q_S^{*} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

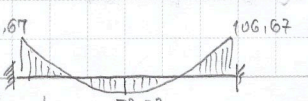
$EJ = 20.000\ kNm^2$; $q = 20\ kN/m'$; $l = 4{,}10\ m$

1° $C_{\varphi 1} = C_{\varphi 2} = \infty$; $C_y = 0$

$q = [0 \quad -0{,}010667 \quad 0 \quad 0]$ (1 2 3 4)

$R_1 = [80 \quad 106{,}67 \quad 0 \quad 53{,}33]$ (6 1 2 3)

$R_2 = [0 \quad -53{,}33 \quad 80 \quad -106{,}67]$ (2 3 7 4)

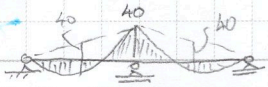


2° $C_{\varphi 1} = C_{\varphi 2} = 0$; $C_y = \infty$

$q = [-0{,}001333 \quad 0 \quad 0 \quad 0{,}001333]$ (1 2 3 4)

$R_1 = [30 \quad 0 \quad 50 \quad -40]$ (6 1 2 3)

$R_2 = [50 \quad 40 \quad 30 \quad 0]$ (2 3 7 4)

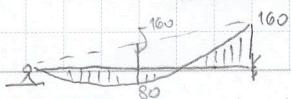


3° $C_{\varphi 1} = 0$; $C_{\varphi 2} = \infty$; $C_y = 0$

$q = [-0{,}010667 \quad -0{,}021333 \quad 0{,}002667 \quad 0]$ (1 2 3 4)

$R_1 = [60 \quad 0 \quad 20 \quad 80]$ (6 1 2 3)

$R_2 = [-20 \quad -80 \quad 100 \quad -160]$ (2 3 7 4)



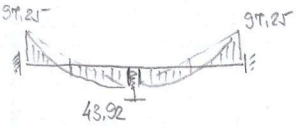
4° $C_{\varphi_1} = C_{\varphi_2} = \infty$

$C_y = 1000$

$q = [0 \ -0{,}0094117 \ 0 \ 0]$ (1 2 3 4)

$R_1 = [75{,}29 \ 97{,}25 \ 4{,}111 \ 43{,}92]$ (6 1 2 3)

$R_2 = [4{,}111 \ -43{,}92 \ 75{,}29 \ -97{,}25]$ (2 3 7 4)



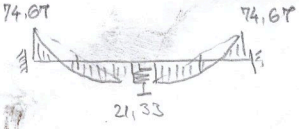
5° $C_{\varphi_1} = C_{\varphi_2} = \infty$

$C_y = 5000$

$q = [0 \ -0{,}0064 \ 0 \ 0]$

$R_1 = [64 \ 74{,}67 \ 16 \ 21{,}33]$ (6 1 2 3)

$R_2 = [16 \ -21{,}33 \ 64 \ -74{,}67]$ (2 3 7 4)



6° $C_{\varphi_1} = C_{\varphi_2} = \infty$

$C_y = 50.000$

$q = [0 \ -0{,}001391 \ 0 \ 0]$

$R_1 = [45{,}22 \ 37{,}10 \ 34{,}78 \ -16{,}23]$ (6 1 2 3)

$R_2 = [34{,}78 \ 16{,}23 \ 45{,}22 \ -37{,}10]$ (2 3 7 4)

7° $C_{\varphi_1} = C_{\varphi_2} = \infty$

$C_y = \infty$

$q = 0$

Sile na krajevima štapova su jednake ekv. opt.