1. ***Zadatak***

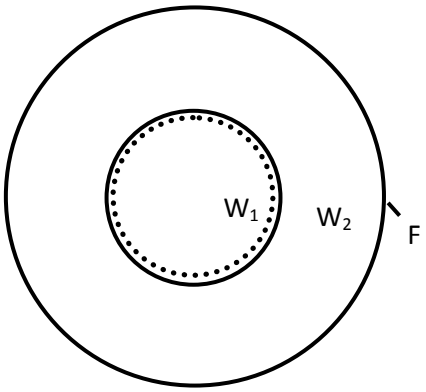
Za datu ploču i opterećenje sračunati i nacrtati dijagrame ugiba W, momenata savijanja Mr i M_φ i transverzalnih sila T_r.

a = 3,4 m

b = 4,6 m

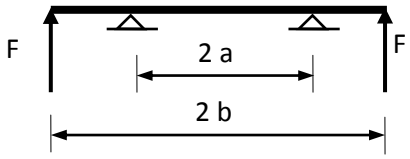
F=16 KN/m

ν = 0,2



Rešenje:

Odabraćemo uporedni poluprečnik takav da je $\rho = \frac{r}{a}$



Rešenje diferencijalnih jednačina ugiba:

Za $a \le r \le b$

Ako na ploči nema površinskog opterećenja onda je rešenje jednačine

$$w_2 = C_1 + C_2\rho^2 + C_3 \ln\rho + C_4\rho^2 \ln\rho .$$

Tražimo izvode po ***r*** koji će nam biti potrebni:

$$\frac{dw_2}{dr} = \frac{1}{a}\left(2C_2\rho + C_3\frac{1}{\rho} + 2C_4\rho\ln\rho + C_4\rho^2\frac{1}{\rho}\right) = \frac{1}{a}\left(2C_2\rho + \frac{C_3}{\rho} + 2C_4\rho\ln\rho + C_4\rho\right)$$

$$\frac{d^2w_2}{dr^2} = \frac{1}{a^2}\left(2C_2 - C_3\rho^{-2} + 2C_4\ln\rho + 2C_4\rho\frac{1}{\rho} + C_4\right) = \frac{1}{a^2}\left(2C_2 - \frac{C_3}{\rho^2} + 2C_4\ln\rho + 3C_4\right)$$

$$\frac{d^3w_2}{dr^3} = \frac{1}{a^3}\left(-(-2)C_3\rho^{-3} + 2C_4\frac{1}{\rho}\right) = \frac{1}{a^3}\left(\frac{2C_3}{\rho^3} + \frac{2C_4}{\rho}\right).$$

Za $0 \le r \le a$

Kako ovaj deo ploče sadrži centar ploče, onda iz uslova o konačnosti ugiba zaključujemo da jednačina ima samo dve nepoznate konstante koje ćemo obeležiti sa *A* i *B*.

Kako na ovom delu ploče nema površinskog opterećenja rešenje jednačine i njegovi izvodi po *r* su:

$$w_1 = A + B\rho^2 .$$

$$\frac{dw_1}{dr} = \frac{1}{a}(2B\rho)$$

$$\frac{d^2 w_1}{dr^2} = \frac{1}{a^2}(2B)$$

$$\frac{d^3 w_1}{dr^3} = 0.$$

Posmatramo desni deo ploče. Imamo dva granična uslova i četiri prelazna uslova.

Granične uslove postavljamo na mestu $r=b$ gde važi w_2:

Za $r = b \Rightarrow \rho = \frac{r}{a} = \frac{b}{a} = \frac{5}{3} = 1{,}353 \Rightarrow$

1) $T_{r,2} = -F$
2) $M_{r,2} = 0$

(na tom mestu ploča ima slobodan kraj i oterećena je silom F koja je negativna).

Prelazne uslove postavljamo na mestu $r=a$ gde važi w_1:

Za $r = a \Rightarrow \rho = \frac{r}{a} = \frac{a}{a} = 1{,}0 \Rightarrow$

3) $w_1 = 0$
4) $w_2 = 0$
5) $\frac{dw_1}{dr} = \frac{dw_2}{dr}$
6) $M_{r,1} = M_{r,2}$

Izrazi za presečne sile:

$$M_r = -k\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r}\right)$$

$$M_{r,1} = -k\left(\frac{1}{a^2}(2B) + \frac{0{,}2}{\rho a}\frac{1}{a}(2B\rho)\right) = -\frac{k}{a^2}2{,}4B$$

$$M_{r,2} = -k\left(\frac{1}{a^2}\left(2C_2 - \frac{C_3}{\rho^2} + 2C_4 \ln\rho + 3C_4\right) + \frac{0{,}2}{\rho a}\frac{1}{a}\left(2C_2\rho + \frac{C_3}{\rho} + 2C_4\rho \ln\rho + C_4\rho\right)\right) =$$

$$= -\frac{k}{a^2}\left(2{,}4C_2 - 0{,}8\frac{C_3}{\rho^2} + 2{,}4C_4 \ln\rho + 3{,}2C_4\right)$$

$$T_r = -k\left(\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right)$$

$$T_{r,1} = -k\left(0 + \frac{1}{\rho a}\cdot\frac{1}{a^2}(2B) - \frac{1}{\rho^2 a^2}\cdot\frac{1}{a}(2B\rho)\right) = 0$$

$$T_{r,2} = -k\left(\frac{1}{a^3}\left(\frac{2C_3}{\rho^3} + \frac{2C_4}{\rho}\right) + \frac{1}{\rho a}\cdot\frac{1}{a^2}\left(2C_2 - \frac{C_3}{\rho^2} + 2C_4 \ln\rho + 3C_4\right) - \right.$$

$$\left. - \frac{1}{\rho^2 a^2}\cdot\frac{1}{a}\left(2C_2\rho + \frac{C_3}{\rho} + 2C_4\rho\ln\rho + C_4\rho\right)\right) =$$

$$= -\frac{k}{a^3}\left(\frac{2C_3}{\rho^3} + \frac{2C_4}{\rho} + \frac{2C_2}{\rho} - \frac{C_3}{\rho^3} + \frac{2C_4\ln\rho}{\rho} + \frac{3C_4}{\rho} - \frac{2C_2\rho}{\rho^2} - \frac{C_3}{\rho^3} - \frac{2C_4\rho\ln\rho}{\rho^2} - \frac{C_4\rho}{\rho^2}\right) = -\frac{k}{a^3}\left(\frac{4C_4}{\rho}\right)$$

Uslovne jednačine za izračunavanje integracionih konstanti:

$\rho = 1{,}353 \Rightarrow$

1) $T_{r,2} = -F \Rightarrow -\frac{k}{a^3}\left(\frac{4C_4}{1{,}353}\right) = -16$

2) $M_{r,2} = 0 \Rightarrow 2{,}4C_2 - 0{,}8\frac{C_3}{1{,}353^2} + 2{,}4C_4 \ln 1{,}353 + 3{,}2C_4 = 0$

$\rho = 1{,}0 \Rightarrow$

3) $w_1 = 0 \Rightarrow A + B = 0$

4) $w_2 = 0 \Rightarrow C_1 + C_2 = 0$

5) $\frac{dw_1}{dr} = \frac{dw_2}{dr} \Rightarrow 2B = 2C_2 + C_3 + C_4$

6) $M_{r,1} = M_{r,2} \Rightarrow 2{,}4B = 2{,}4C_2 - 0{,}8C_3 + 3{,}2C_4$

Vrednosti konstanti:

$$A = \frac{96{,}48}{k}; B = -\frac{96{,}48}{k}; C_1 = \frac{309{,}193}{k}; C_2 = -\frac{309{,}193}{k}; C_3 = C_4 = \frac{212{,}713}{k}$$

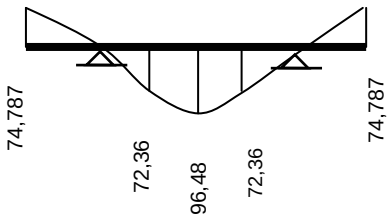
Sračunavanje ugiba

$$w_1 = A + B\rho^2 = \frac{96{,}48}{k} - \frac{96{,}48}{k}\rho^2$$

$kw_{1,\rho=0} = 96{,}48$

$kw_{1,\rho=0{,}5} = 72{,}36$

$kw_{1,\rho=1} = 0$



Dijagram ugiba k*w*

$$w_2 = C_1 + C_2\rho^2 + C_3\ln\rho + C_4\rho^2\ln\rho = \frac{309{,}193}{k} - \frac{309{,}193}{k}\rho^2 + \frac{212{,}713}{k}\ln\rho + \frac{212{,}713}{k}\rho^2\ln\rho$$

$kw_{2,\rho=1} = 0$

$kw_{2,\rho=1{,}353} = 74{,}787$

Sračunavanje momenta M_r:

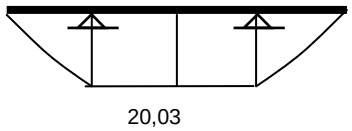
$$M_{r,1} = -\frac{k}{a^2} 2{,}4B$$

$$M_{r1,\rho=0} = M_{r1,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{r,2} = -\frac{k}{a^2}\left(2{,}4C_2 - 0{,}8\frac{C_3}{\rho^2} + 2{,}4C_4 \ln\rho + 3{,}2C_4\right)$$

$$M_{r2,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{r2,\rho=1{,}353} = 0$$



Dijagram momenata savijanja M_r [kNm/m]

Sračunavanje momenta M_r:

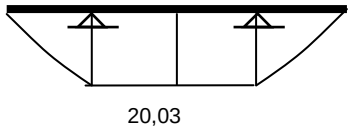
$$M_{r,1} = -\frac{k}{a^2} 2{,}4B$$

$$M_{r1,\rho=0} = M_{r1,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{r,2} = -\frac{k}{a^2}\left(2{,}4C_2 - 0{,}8\frac{C_3}{\rho^2} + 2{,}4C_4 \ln\rho + 3{,}2C_4\right)$$

$$M_{r2,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{r2,\rho=1{,}353} = 0$$



Dijagram momenata savijanja M_r [kNm/m]

Sračunavanje momenta M_φ:

$$M_\varphi = -k\left(\nu\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

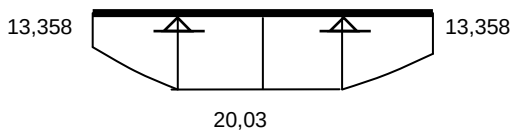
$$M_{\varphi,1} = -\frac{k}{a^2} 2{,}4B$$

$$M_{\varphi1,\rho=0} = M_{r1,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{\varphi,2} = -\frac{k}{a^2}\left(2{,}4C_2 + 0{,}8\frac{C_3}{\rho^2} + 2{,}4C_4 \ln\rho + 1{,}6C_4\right)$$

$$M_{\varphi2,\rho=1} = 20{,}03 \ kNm/m$$

$$M_{\varphi2,\rho=1{,}353} = 13{,}358 \ kNm/m$$



Dijagram momenata savijanja M_φ [kNm/m]

Sračunavanje transverzalnih sila

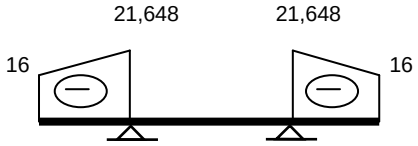
$$T_r = -k\left(\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right)$$

$$T_{r1} = 0$$

$$T_{r2} = -\frac{k}{a^3}\left(\frac{4C_4}{\rho}\right)$$

$$T_{r2,\rho=1} = -21{,}648 \ kN$$

$$T_{r2,\rho=1{,}353} = -16 \ kN$$



Dijagram transverzalnih sila T_r [kN]

Kontrola: $A = \dfrac{F \cdot 2b\pi}{2a\pi} = 21{,}648$