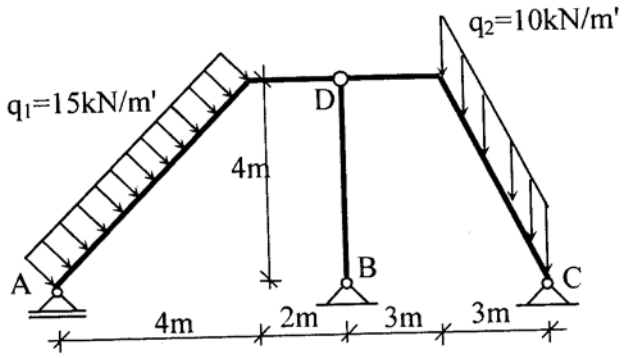
Vežba br. 8b

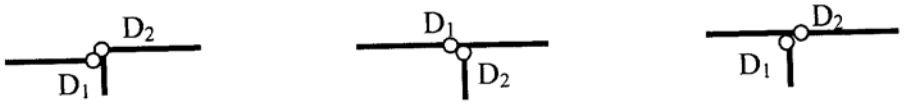
Složen zglob

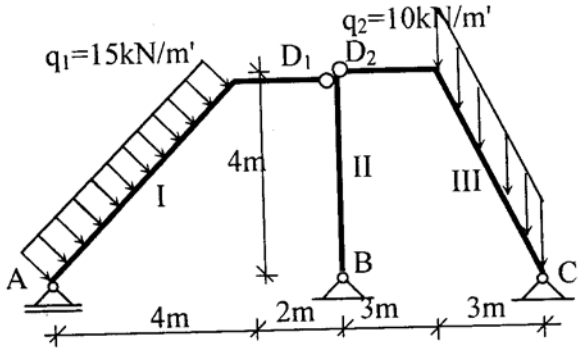
Odrediti reakcije veza.

1.



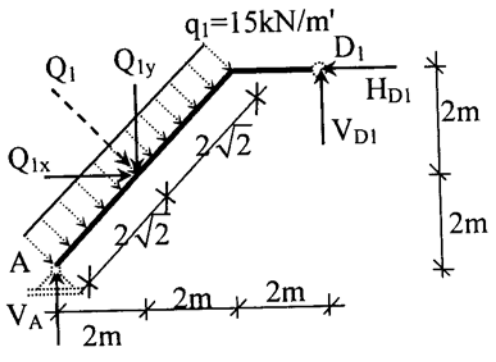
Konstrukcija se sastoji od tri tela, koja su vezana složenim zglobom u D. Pri rešavanju zadataka složen zglob se posmatra kao dva prosta zgloba. Na slici su prikazana tri varijantna rešenja. Usvojeno je prvo od prikazana tri, kod koga je cilindričnim zglobom D_1 ostvarena veza između tela I i tela II, a cilindričnim zglobom D_2 vezana tela II i III.
Detalji složenog zgloba:





Sistem je statički određen jer je broj nepoznatih komponenti reakcija veza jednak broju uslovnih jednačina ravnoteže:
n=9 (V_A, V_B, H_B, V_C, H_C, V_{D1}, H_{D1}, V_{D2}, H_{D2}), r=3t=3·3=9, $\Rightarrow$ n=r.

Telo I



$$Q_1 = q1 = 15 \cdot 4\sqrt{2} = 60\sqrt{2} = 84.85\,\text{kN},$$

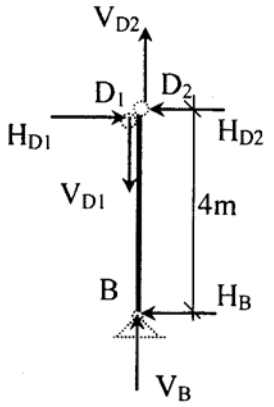
$$Q_{1x} = Q_1 \cos 45° = 60\sqrt{2}\,\frac{\sqrt{2}}{2} = 60\text{kN},$$

$$Q_{1y} = Q_1 \sin 45° = 60\sqrt{2}\,\frac{\sqrt{2}}{2} = 60\text{kN}.$$

Jednačine ravnoteže za telo I:

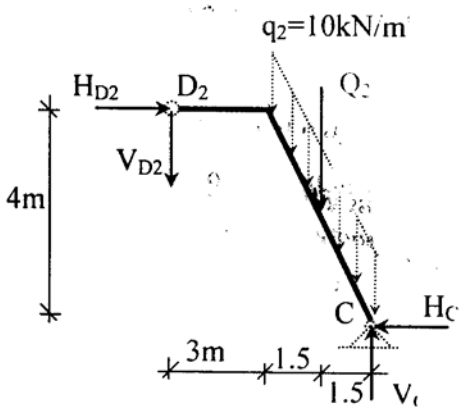
$$\sum X = 0 \Rightarrow -H_{D1} + Q_{1x} = 0, \quad (1)$$
$$\sum Y = 0 \Rightarrow V_{D1} + V_A - Q_{1y} = 0, \quad (2)$$
$$\sum M_D = 0 \Rightarrow -V_A \cdot 6 + Q_{1y} \cdot 4 + Q_{1x} \cdot 2 = 0, \quad (3)$$
$$(1) \Rightarrow H_{D1} = Q_{1x} = 60\text{kN}, \quad (3) \Rightarrow V_A = 40\text{kN}. \quad (2) \Rightarrow V_{D1} = 60 - V_A = 20\text{kN}$$

Jednačine ravnoteže za telo II:



$$\sum X = 0 \Rightarrow -H_{D2} + H_{D1} - H_B = 0, \quad (4)$$
$$\sum Y = 0 \Rightarrow V_{D2} - V_{D1} + V_B = 0, \quad (5)$$
$$\sum M_D = 0 \Rightarrow H_B \cdot 4 = 0, \quad (6),$$
$$(6) \Rightarrow H_B = 0, \quad (4) \Rightarrow H_{D2} = H_{D1} = 60\text{kN}.$$

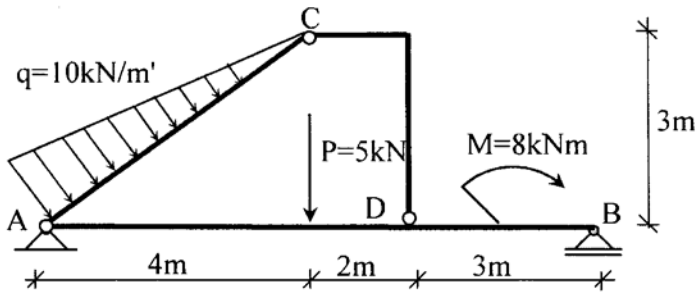
Jednačine ravnoteže za telo III:



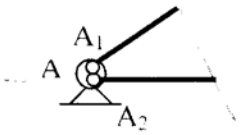
$$Q_2 = q\,L = 10 \cdot \sqrt{3^2 + 4^2} = 10 \cdot \sqrt{25} = 50\text{kN},$$
$$\sum X = 0 \Rightarrow H_{D2} - H_C = 0, \quad (7)$$
$$\sum Y = 0 \Rightarrow V_C - V_{D2} - Q_2 = 0, \quad (8),$$
$$\sum M_C = 0 \Rightarrow V_{D2} \cdot 6 - H_{D2} \cdot 4 + Q_2 \cdot 1.5 = 0, \quad (9)$$
$$(7) \Rightarrow H_C = H_{D2} = 60\text{kN},$$
$$(9) \Rightarrow 6V_{D2} = H_{D2} \cdot 4 - Q_2 \cdot 1.5, \Leftrightarrow V_{D2} = 27.5\text{kN},$$
$$(8) \Rightarrow V_C = V_{D2} + Q_2 = 77.5\text{kN}.$$

$$(5) \Rightarrow V_B = -7.5\text{kN}.$$

2.

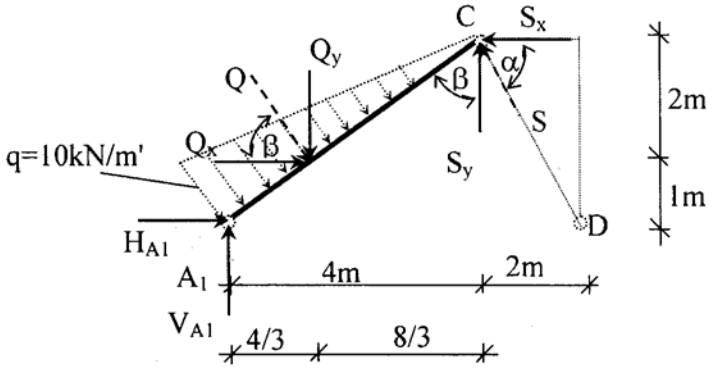


Detalj složenog zgloba A:



Broj nepoznatih reakcija veza:
n=8, (V_A, H_A, V_B, V_{A1}, H_{A1}, V_{A2}, H_{A2}, S), (telo CD je prosto, pa su reakcije u C i D istog intenziteta i pravca, a suprotnog smera, obeležene u daljem radu sa S),
broj jednačina ravnoteže:
r=3t+2=3·2+2=8, (telo A_1C, telo A_2B i dve jednačine ravnoteže sistema sila u čvoru A),
n=r ⇒ sistem je statički određen.

Telo I

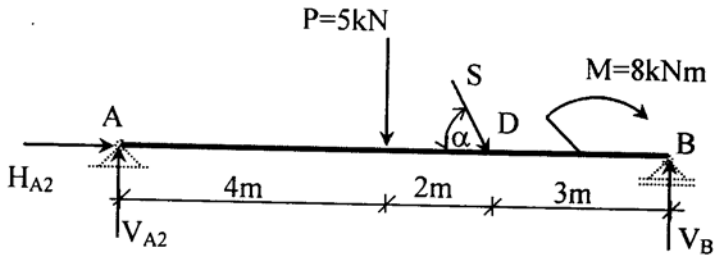


$$\cos\alpha=\frac{2}{\sqrt{13}},\quad \sin\alpha=\frac{3}{\sqrt{13}},$$
$$S_x=S\cos\alpha,\quad S_y=S\sin\alpha,$$
$$Q=\frac{ql}{2}=\frac{10\cdot5}{2}=25\text{kNm},$$
$$\cos\beta=\frac{3}{5},\quad \sin\beta=\frac{4}{5},$$
$$Q_x=Q\cos\beta=15\text{kN},$$
$$Q_y=Q\sin\beta=20\text{kN}.$$

Jednačine ravnoteže za telo I:

$$\sum X=0\Rightarrow H_{A1}-S_x+Q_x=0,\quad (1),$$
$$\sum Y=0\Rightarrow V_{A1}+S_y-Q_y=0,\quad (2).$$
$$\sum M_A=0\Rightarrow S_y\cdot4+S_x\cdot3-Q\cdot\frac{5}{3}=0,\quad (3)$$
$$(3)\Rightarrow S=8.35\text{kN},\quad (2)\Rightarrow V_{A1}=13.06\text{kN},\quad (1)\Rightarrow H_{A1}=-10.37\text{kN}.$$

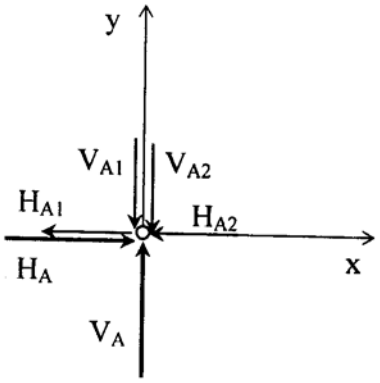
Telo II



Jednačine ravnoteže za telo II:

$$\sum X = 0 \Rightarrow H_{A2} + S_x = 0, \quad (4),$$
$$\sum Y = 0 \Rightarrow V_{A2} + V_B - P - S_y = 0, \quad (5),$$
$$\sum M_A = 0 \Rightarrow V_B \cdot 9 - P \cdot 4 - S_y \cdot 6 - M = 0, \quad (6),$$
$$(4) \Rightarrow H_{A2} = -4.63\text{kN},$$
$$(6) \Rightarrow V_B = 7.74\text{kN},$$
$$(5) \Rightarrow V_{A2} = 4.2\text{kN}.$$

Ravnoteža čvora A:



$$\sum X = 0 \Rightarrow -H_{A2} - H_{A1} + H_A = 0, \quad (7)$$
$$\sum Y = 0 \Rightarrow -V_{A2} - V_{A1} + V_A = 0, \quad (8)$$
$$(7) \Rightarrow H_A = H_{A1} + H_{A2} = -4.63 - 10.37 = -15\text{kN},$$
$$(8) \Rightarrow V_A = V_{A1} + V_{A2} = 13.06 + 4.20 = 17.26\text{kN}.$$