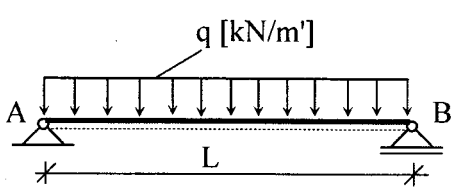
Vežba br. 12a

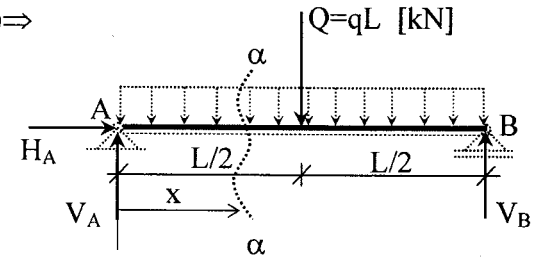
Sile u presecima. Prosta greda opterećena kontinualnim opterećenjem

Odrediti sile u presecima za datu prostu gredu i nacrtati dijagrame momenata savijanja, transverzalnih sila i normalnih sila.

1.



(A5)⇒

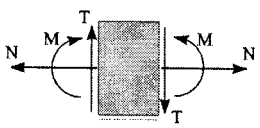


Uslovne jednačine ravnoteže:

$$\sum X = 0 \Rightarrow H_A = 0, \quad (1)$$

$$\sum Y = 0 \Rightarrow V_A + V_B - Q = 0, \quad (2)$$

$$\sum M_A = 0 \Rightarrow V_B \cdot L - Q \cdot \frac{L}{2} = 0, \quad (3)$$



$$(1) \Rightarrow H_A = 0, \quad (3) \Rightarrow V_B = \frac{Q}{2} = \frac{qL}{2}, \quad (2) \Rightarrow V_A = \frac{Q}{2} = \frac{qL}{2}.$$

Analitički izrazi za moment savijanja, transverzalnu i normalnu silu na prostoj gredi AB (presek α – α), gde se x kreće u granicama od nula do L, glase:

$$M(x) = V_A x - q x \frac{x}{2} = \frac{qL}{2} x - q \frac{x^2}{2},$$

$$T(x) = V_A - q x = \frac{qL}{2} - q x,$$

$$N(x) = 0,$$

a vrednosti momenata savijanja, transverzalnih i normalnih sila u krajnjim tačkama A i B su:

$$\text{za } x = 0 \rightarrow M_A = 0, \; T_A = \frac{qL}{2}, \; N_A = 0,$$

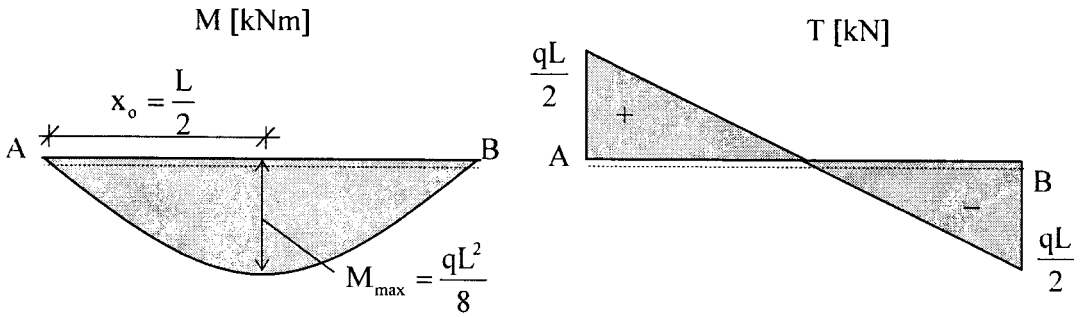
$$\text{za } x = L \rightarrow M_B = 0, \; T_B = -\frac{qL}{2}, \; N_B = 0.$$

Moment savijanja se menja po zakonu parabole drugog reda, čija strela može da se odredi iz uslova ekstremuma funkcije. Imajući u vidu diferencijalnu zavisnost između momenta savijanja i transverzalne sile u proizvoljnom preseku grednog nosača, maksimalni moment savijanja se nalazi u preseku u kome je transverzalna sila jednaka nuli:

$$T(x) = \frac{dM(x)}{dx} = 0 \quad \Rightarrow \quad \frac{q}{2}(L - 2x_o) = 0 \quad \Rightarrow \quad x_o = \frac{L}{2}.$$

Veličina tog maksimuma je:

$$M_{max} = M_{\left(x_o=\frac{L}{2}\right)} = \frac{qL}{2}\frac{L}{2} - q\frac{\left(\frac{L}{2}\right)^2}{2} = \frac{qL^2}{8}.$$



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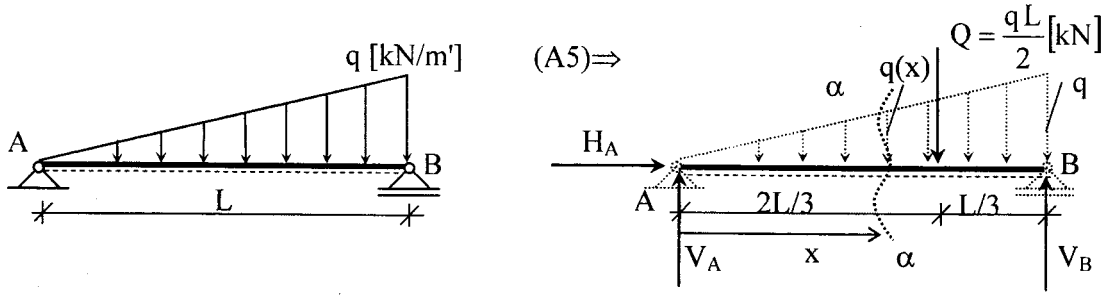
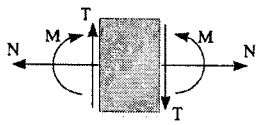


$$\sum X = 0 \Rightarrow H_A = 0, \quad (1)$$

$$\sum Y = 0 \Rightarrow V_A + V_B - Q = 0, \quad (2)$$

$$\sum M_A = 0 \Rightarrow V_B \cdot L - Q \cdot \frac{2L}{3} = 0, \quad (3)$$



$$(1) \Rightarrow H_A = 0, \quad (3) \Rightarrow V_B = \frac{2Q}{3} = \frac{qL}{3}, \quad (2) \Rightarrow V_A = \frac{Q}{3} = \frac{qL}{6}.$$

Pri formiranju analitičkih izraza za momente savijanja i transverzalne sile potrebno je poznavati zakon promene kontinualnog opterećenja q=q(x) na rastojanju x od oslonca A. U ovom primeru to je prava linija, a njena jednačina se može odrediti pomoću sličnosti pravouglih trouglova iz kojih sledi proporcionalnost odgovarajućih stranica:

$$q : q(x) = L : x \quad \Rightarrow q(x) = \frac{qx}{L}.$$

Analitički izraz za moment savijanja, gledajući levo od preseka i levi deo šablona, glasi:

$$M(x) = V_A x - \frac{q(x)x}{2}\frac{x}{3} = \frac{qL}{6}x - \frac{\frac{qx}{L}x}{2}\frac{x}{3} = \frac{qL}{6}x - \frac{q}{6L}x^3.$$

a vrednosti momenata savijanja na početku i na kraju polja su:

za $x = 0 \rightarrow M_A = 0,$

za $x = L \rightarrow M_B = 0.$

Prvi izvod momenta savijanja po koordinati x je jednak transverzalnoj sili:

$$T(x)=\frac{dM(x)}{dx}=\frac{qL}{6}-\frac{q}{2L}x^2,$$

a vrednosti transverzalnih sila na početku i na kraju polja su:

$$x=0 \rightarrow T_A=\frac{qL}{6},$$

$$x=L \rightarrow T_B=-\frac{qL}{3}.$$

Kako transverzalna sila na jednom delu polja ima pozitivan, a na drugom negativan znak, to ukazuje na to da u polju AB postoji presek u kome je transverzalna sila jednak nuli. Ako je taj presek na rastojanju x_o od oslonca A, može se napisati:

$$\frac{qL}{6}-\frac{q}{2L}x_o^2=0,$$

odakle se određuje položaj traženog preseka :

$$x_o=\frac{L\sqrt{3}}{3}.$$

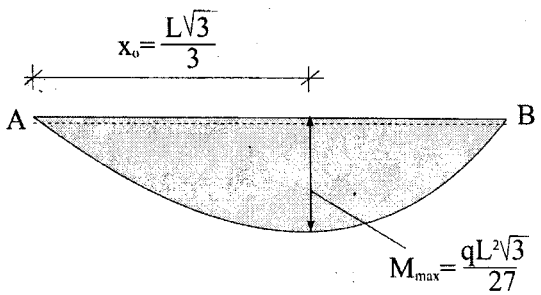
Pošto je:

$$T(x)=\frac{dM(x)}{dx},$$

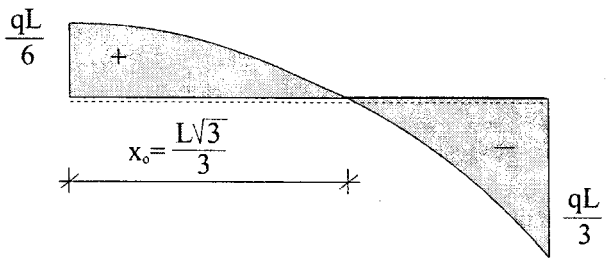
u tački $x=x_o$ funkcija M(x) ima prevojnu tačku, to jest u tom preseku moment savijanja ima maksimalnu vrednost:

$$M_{max}=M_{\left(x_o=\frac{L\sqrt{3}}{3}\right)}=\frac{qL}{6}\frac{L\sqrt{3}}{3}-\frac{q}{6L}\left(\frac{L\sqrt{3}}{3}\right)^2=\frac{qL^2\sqrt{3}}{27}.$$

Aksijalne sile su na celoj gredi jednake nuli.

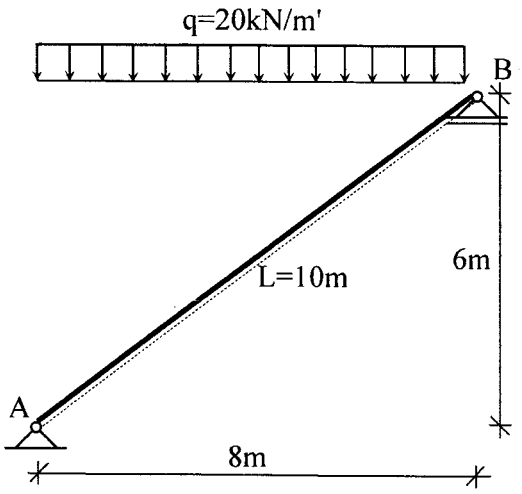


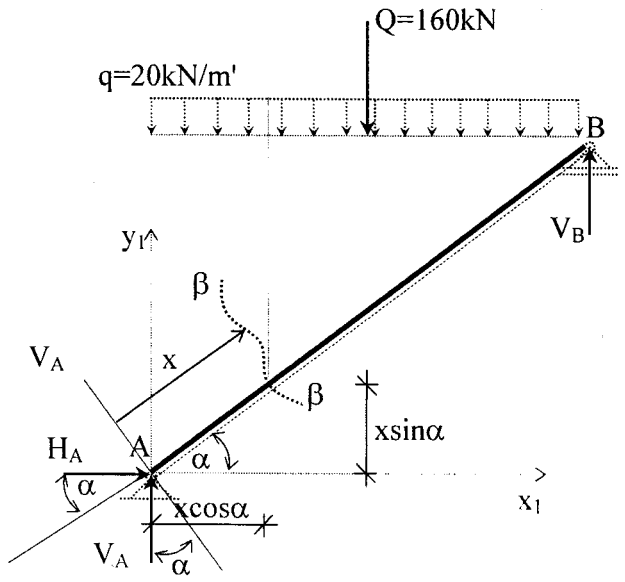
M [kNm]



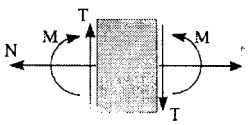
T [kN]

3.





$$L=\sqrt{8^2+6^2}=10\text{m},\quad \sin\alpha=\frac{6}{10}=0.60,\quad \cos\alpha=\frac{8}{10}=0.80,$$



$$Q=qL\cos\alpha=160\text{kN}.$$

Reakcije veza:

$$\sum X_1=0\Rightarrow H_A=0,\quad (1)$$

$$\sum Y_1=0\Rightarrow V_A+V_B-Q=0,\quad (2)$$

$$\sum M_A=0\Rightarrow V_B\cdot 8-Q\cdot 4=0,\quad (3)$$

$$(1)\Rightarrow H_A=0,\quad (3)\Rightarrow V_B=\frac{Q}{2}=\frac{qL\cos\alpha}{2}=80\text{kN},\quad (2)\Rightarrow V_A=\frac{Q}{2}=\frac{qL\cos\alpha}{2}=80\text{kN}.$$

Analitički izrazi za moment savijanja, transverzalnu i aksijalnu silu u polju proste grede su

$$M_\beta=V_A\,x\cos\alpha-Q(x)\frac{x\cos\alpha}{2}=\frac{qL\cos\alpha}{2}x\cos\alpha-q\,x\cos\alpha\frac{x\cos\alpha}{2}=\frac{qL\cos\alpha}{2}x\cos\alpha-\frac{q\cos^2\alpha}{2}x^2.$$

$$T_\beta=V_A\cos\alpha-q\,x\cos\alpha\cos\alpha=\frac{qL\cos\alpha}{2}\cos\alpha-q\,x\cos^2\alpha=\frac{qL}{2}\cos^2\alpha-q\,x\cos^2\alpha.$$

$$N_\beta=-V_A\sin\alpha+q\,x\cos\alpha\sin\alpha=-\frac{qL\cos\alpha}{2}\sin\alpha+q\,x\cos\alpha\sin\alpha.$$

Položaj i vrednost maksimalnog momenta u polju AB:

$$\frac{dM}{dx}=T=A\cos\alpha-q\,x_o\cos^2\alpha=0\quad\Rightarrow\quad x_o=\frac{qL\cos^2\alpha}{2q\cos^2\alpha}=\frac{L}{2}=5\text{m}.$$

$$M_{max}=\frac{qL\cos^2\alpha}{2}\frac{L}{2}-\frac{q}{2}\frac{L^2}{4}\cos^2\alpha=\frac{qL^2}{8}\cos^2\alpha=\frac{20\cdot 10^2}{8}\left(\frac{8}{10}\right)^2=160\text{kNm}.$$

Vrednosti momenata savijanja, transverzalnih sila i aksijalnih sila na početku i na kraju polja su:

$$M_A = 0, \quad M_B = 0,$$

$$T_A = V_A \cos\alpha = 80\frac{4}{5} = 64\text{kN},$$

$$T_B = \frac{qL}{2}\cos^2\alpha - qL\cos^2\alpha = -\frac{qL}{2}\cos^2\alpha$$

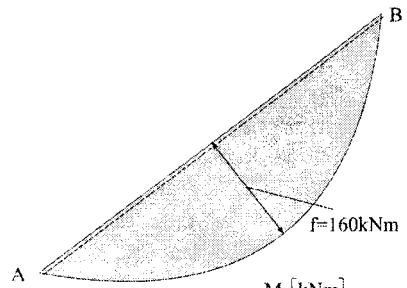
$$= -V_B \cos\alpha = -80\frac{4}{5} = -64\text{kN},$$

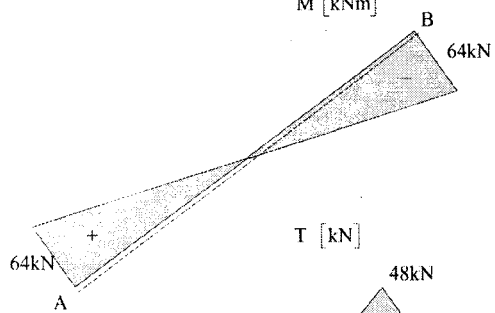
$$N_A = -\frac{qL\cos\alpha}{2}\sin\alpha = -V_A \sin\alpha$$

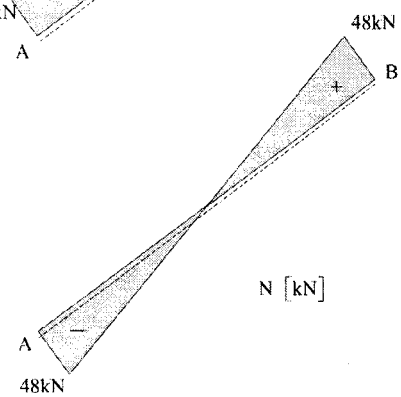
$$= -80\frac{6}{10} = -48\text{kN},$$

$$N_B = -\frac{qL\cos\alpha}{2}\sin\alpha + qL\cos\alpha\sin\alpha$$

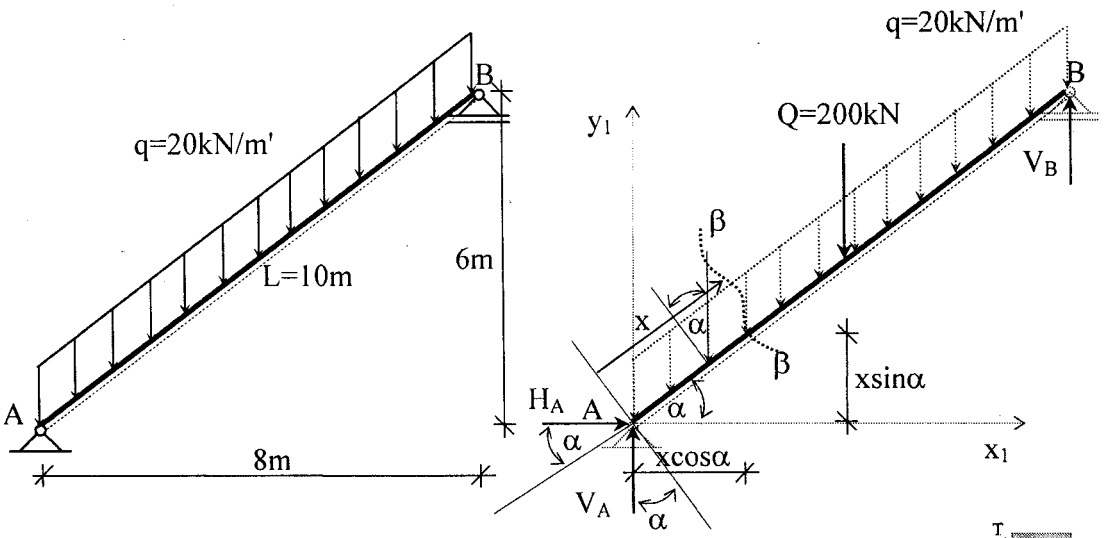
$$= \frac{qL\cos\alpha}{2}\sin\alpha = V_B \sin\alpha = 80\frac{6}{10} = 48\text{kN}.$$

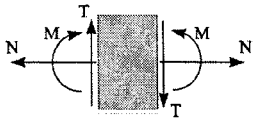






4.





Reakcije veza:

$$\sum X_1 = 0 \Rightarrow H_A = 0, \quad (1)$$

$$\sum Y_1 = 0 \Rightarrow V_A + V_B - Q = 0, \quad (2)$$

$$\sum M_A = 0 \Rightarrow V_B \cdot 8 - Q \cdot 4 = 0, \quad (3)$$

$$(1) \Rightarrow H_A = 0, \quad (3) \Rightarrow V_B = \frac{Q}{2} = \frac{qL}{2} = 100\text{kN}, \quad (2) \Rightarrow V_A = \frac{Q}{2} = \frac{qL}{2} = 100\text{kN}.$$

Analitički izrazi za moment savijanja, transverzalnu i normalnu silu u polju proste grede glase:

$$M_\beta = V_A x \cos\alpha - (q\cos\alpha) x \frac{x}{2} = \frac{qL}{2} x \cos\alpha - \frac{q x^2}{2} \cos\alpha.$$

$$T_\beta = V_A \cos\alpha - (q\cos\alpha) x = \frac{qL}{2} \cos\alpha - q x \cos\alpha,$$

$$N_\beta = -V_A \sin\alpha + (q\sin\alpha) x = -\frac{qL}{2} \sin\alpha + q x \sin\alpha.$$

Položaj i vrednost maksimalnog momenta u polju AB:

$$\frac{dM}{dx} = T = V_A \cos\alpha - q x_o \cos\alpha = 0$$

$$\Rightarrow \quad x_o = \frac{qL\cos\alpha}{2q\cos\alpha} = \frac{L}{2} = 5\text{m},$$

$$M_{max} = \frac{qL}{2}\frac{L}{2}\cos\alpha - \frac{q\left(\frac{L}{2}\right)^2}{2}\cos\alpha = \frac{qL^2}{8}\cos\alpha$$

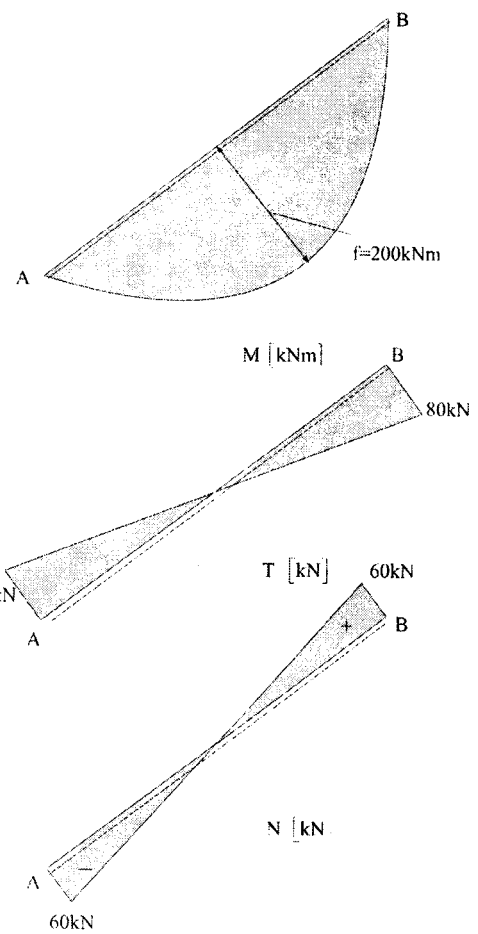
$$= \frac{20 \cdot 10^2}{8}\frac{8}{10} = 200\text{kNm}.$$

Vrednosti momenata savijanja, transverzalnih sila i aksijalnih sila na početku i na kraju polja su:

$M_A=0, \quad M_B=0,$

$$T_A = -T_B = V_A \cos\alpha = \frac{qL}{2}\cos\alpha = 100 \cdot \frac{8}{10} = 80\text{kN}.$$

$$-N_A = N_B = V_A \cos\alpha = 100\frac{6}{10} = 60\text{kN}.$$



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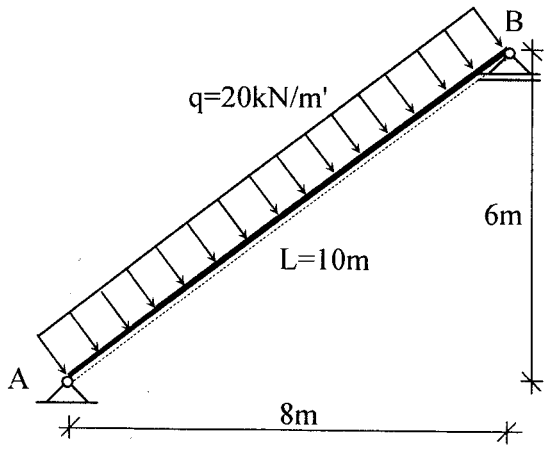
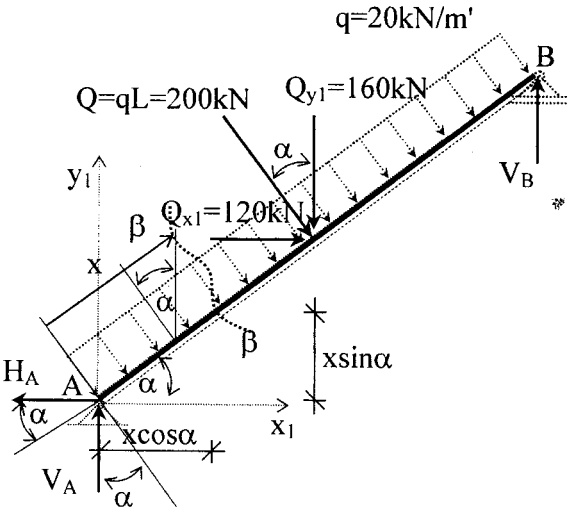
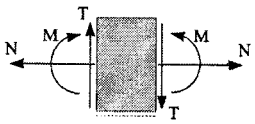




$$\sin\alpha = \frac{6}{10} = 0.60 \qquad \cos\alpha = \frac{8}{10} = 0.80$$



Reakcije veza:

$$\sum X_1 = 0 \Rightarrow -H_A + Q\sin\alpha = 0, \quad (1)$$

$$\sum Y_1 = 0 \Rightarrow V_A + V_B - Q\cos\alpha = 0, \quad (2)$$

$$\sum M_A = 0 \Rightarrow V_B \cdot 8 - Q\cdot 5 = 0, \quad (3) \quad \text{ili} \quad V_B\cos\alpha\cdot L - qL\cdot\frac{L}{2} = 0$$

$$(1) \Rightarrow H_A = 120\text{kN}, \quad \text{ili} \quad H_A = qL\sin\alpha,$$

$$(3) \Rightarrow V_B = \frac{5}{8}Q = 125\text{kN}, \quad \text{ili} \quad V_B = \frac{qL}{2\cos\alpha},$$

$$(2) \Rightarrow V_A = Q\cos\alpha - V_B = 35\text{kN}, \quad \text{ili} \quad V_A = qL\cos\alpha - \frac{qL}{2\cos\alpha}.$$

Analitički izrazi za moment savijanja, transverzalnu i aksijalnu silu u polju proste grede glase:

$$M_\beta = V_A\, x\cos\alpha + H_A\, x\sin\alpha - \frac{q\,x^2}{2},$$

$$T_\beta = V_A\cos\alpha + H_A\sin\alpha - q\,x,$$

$$N_\beta = H_A\cos\alpha - V_A\sin\alpha.$$

Položaj i vrednost maksimalnog momenta u polju AB:

$$\frac{dM}{dx} = T = V_A\cos\alpha - H_A\sin\alpha - q\,x_o = 0 \quad \Rightarrow \quad x_o = \frac{L}{2} = 5\text{m},$$

$$M_{max} = 35\frac{8}{10}\cdot 5 + 120\frac{3}{5}\cdot 5 - \frac{20\cdot 5^2}{2} = 250\text{kNm}.$$

Vrednosti momenata savijanja, transverzalnih sila i aksijalnih sila na početku i na kraju polja su:

M_A=0, M_B=0.

$$T_A = 35 \cdot \frac{8}{10} + 120 \cdot \frac{3}{5} = 100\text{kN},$$

$$T_B = 35 \cdot \frac{8}{10} + 120 \cdot \frac{3}{5} - 20 \cdot 10 = -100\text{kN},$$

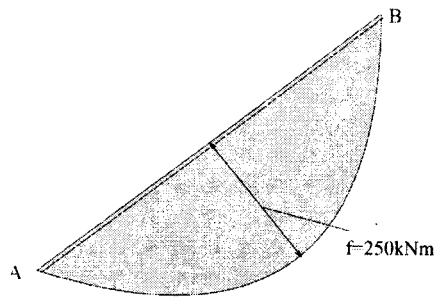
$$N_A = 120 \cdot \frac{8}{10} - 35 \cdot \frac{6}{10} = 75\text{kN},$$

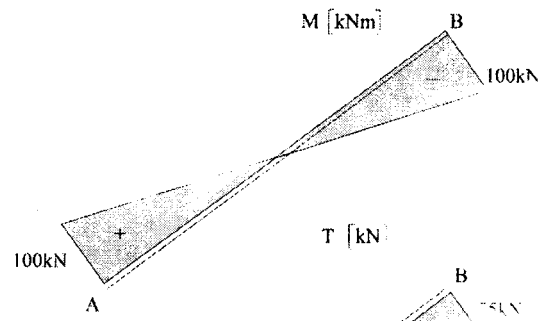
$$N_B = 120 \cdot \frac{8}{10} - 35 \cdot \frac{6}{10} = 75\text{kN}.$$

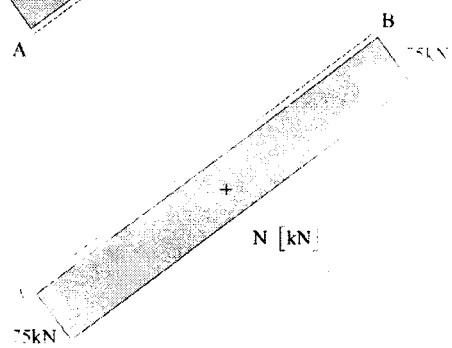
Maksimalni moment savijanja u polju izražen preko vrednosti kontinualnog opterećenja q i dužine grede L je:

$$M_{max} = V_B \cos\alpha (L - x_o) - q(L - x_o)\frac{L - x_o}{2}$$

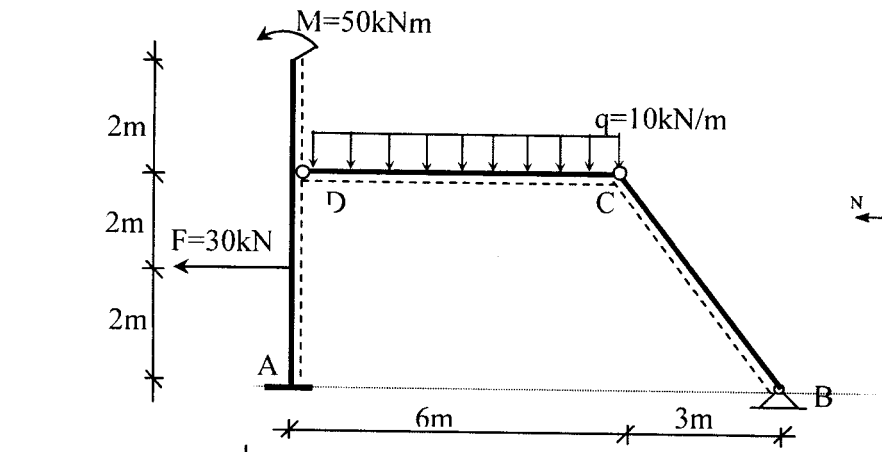
$$= \frac{qL}{2\cos\alpha}\cos\alpha\left(L - \frac{L}{2}\right) - \frac{q}{2}\left(L - \frac{L}{2}\right)^2 = \frac{qL^2}{8}.$$

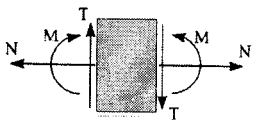


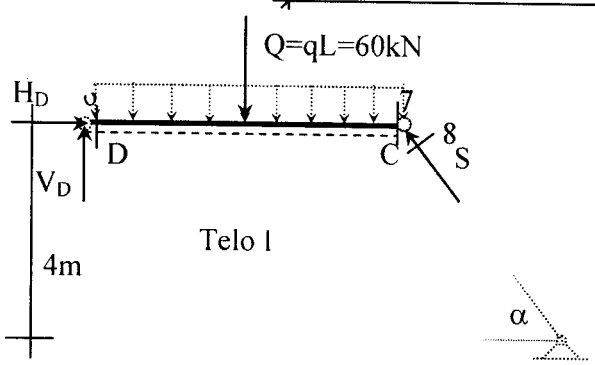


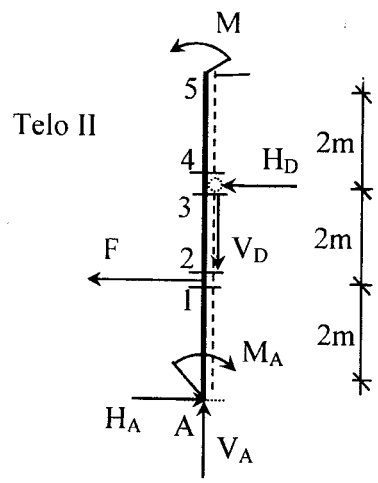


6. Odrediti komponente reakcija veza za dati sistem i opterećenje. Sračunati sile u presecima i nacrtati dijagrame M, T i N.









Jednačine ravnoteže za telo I:

$$\sum X = 0 \Rightarrow H_D - S\cos\alpha = 0, \quad (1)$$
$$\sum Y = 0 \Rightarrow V_D - Q + S\sin\alpha = 0, \quad (2)$$
$$\sum M_D = 0 \Rightarrow S\sin\alpha \cdot 6 - Q \cdot 3 = 0, \quad (3)$$
$$(1) \Rightarrow H_D = 22.5\text{kN}, \quad (3) \Rightarrow S = 37.5\text{kN}, \quad (2) \Rightarrow V_D = 30\text{kN}.$$

Jednačine ravnoteže za telo II:

$$\sum X = 0 \Rightarrow -H_D + H_A - F = 0, \quad (4)$$
$$\sum Y = 0 \Rightarrow -V_D + V_A = 0, \quad (5)$$
$$\sum M_A = 0 \Rightarrow -M_A + M + F \cdot 2 + H_D\, 4 = 0, \quad (6)$$
$$(4) \Rightarrow H_A = 52.5\text{kN}, \quad (6) \Rightarrow M_A = 200\text{kN}, \quad (5) \Rightarrow V_A = 30\text{kN}.$$

Sile u presecima

Momenti savijanja:

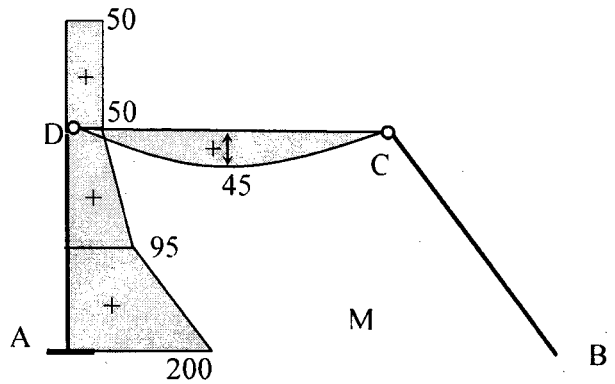
$M_B = M_C = M_D = 0,$

$M_5 = M = 50\text{kNm} = M_4 = M_3,$

$M_1 = M + H_D \cdot 2 = 95\text{kNm} = M_2.$

$M_A = 200\text{kNm}.$

$$f = \frac{qL^2}{8} = \frac{10 \cdot 6^2}{8} = 45\text{kNm}.$$



Transverzalne sile:

$T_B = 0 = T_8,$

$T_7 = -S\sin\alpha = -30\text{kN},$

$T_6 = -S\sin\alpha + Q = 30\text{kN},$ ili $T_6 = V_D = 30\text{kN},$

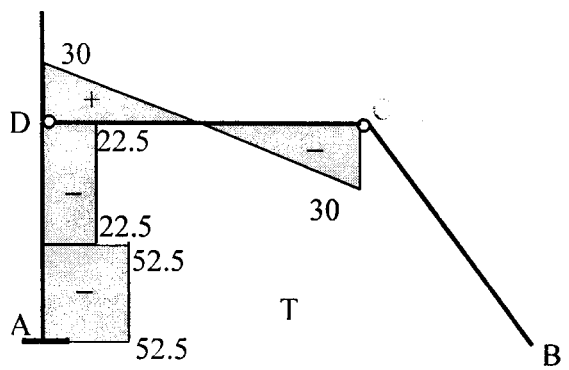
$T_5 = 0 = T_4,$

$T_3 = -H_D = -22.5\text{kN},$

$T_2 = -H_D = -22.5\text{kN},$

$T_1 = -H_D - F = -22.5 - 30 = -52.5\text{kN},$

$T_A = -H_A = -52.5\text{kN}.$



Normalne sile:

$N_8 = -S = -37.5\text{kN},$

$N_7 = -S\cos\alpha = -22.5\text{kN} = N_6,$

$N_5 = 0 = N_4.$

$N_3 = -V_D = -30\text{kN} = N_2 = N_1 = N_A.$